EPIMENIDES AT THE COMPUTER*

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If "computer science" is indeed a science, it is in part because the languages in which the programmer communicates with the computer are akin to axiomatized formal systems, such as the propositional calculus or the number-theory system of Russell and Whitehead's *Principia Mathematica*. Even though a set of statements in such a formal system is ordinarily a proof, while a set of statements in a computer language is ordinarily a routine that instructs the computer to execute a sequence of logical steps, certain central concepts of mathematical logic have been shown to be directly relevant to computer programming.

One such concept is that of recursive definition. A recursive function, for example, the definition of a series of numbers \(G(0), G(1), \ldots G(n)\) ...

\[
G(n) = n - G(G(n-1)) \text{ for } n > 0 \\
G(0) = 0
\]

cannot be evaluated for an arbitrary value of \(n\) by conventional algebraic techniques because one cannot immediately compute \(G(n-1)\), let alone \(G(G(n-1))\). But, knowing that \(G(0) = 0\), one can determine that for \(n = 1\), \(G(n-1) = 0\), so \(G(G(n-1)) = 0\), and so \(G(n) = 1\). Knowing that \(G(n) = 1\) for \(n = 1\), one can, by performing a series of iterative calculations, each depending on the result of its predecessor, evaluate \(G(n)\) for \(n = 2, 3, \ldots\) until the required value of \(n\) is reached. Such calculations, tedious and error-prone when carried out by a human being, are just what computers are good at, and an experienced programmer will try to cast the problem he wishes to solve in the form of a recursive definition.

A closely related notion is that of nested logical structure. An extremely complex proof in the propositional calculus can be made perspicuous if it can be organized as a group of subordinate derivations, and a subordinate derivation may in turn have subordinate derivations of its own, the process extending to whatever depth of nesting is required (obviously, if the form of each successively nested derivation is the same, the proof is simply recursive). Analogously, a computer program can be organized as a series of calls to subroutines, each of which may in turn call other subroutines, and so on.

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Acknowledgment: Support from NICHD Grant HD01994, National Institutes of Health, is gratefully acknowledged.

[HASKINS LABORATORIES: Status Report on Speech Research SR-59/60 (1979)]
Again, logicians have long been interested in the logical status of self-referent sentences because they can be paradoxical (e.g., Epimenides the Cretan's assertion that "all Cretans are liars," or in other words, "this statement is false"). Not all self-referent statements in natural language lead to paradox, however (e.g., "This statement is in English") and all computer languages of practical interest allow self-reference, because this property permits a program to modify itself as the computation proceeds. Part of the routine for executing the nth step of the evaluation of a recursive function is a slight modification that converts it into a routine for executing the n+1st step. It is the property of self-reference in the programming language that essentially distinguishes a computer from a non-programmable calculator.

The programmer, however, must be very careful with recursive procedures, nested structures and self-reference. If the number of iterations or the initial conditions of a recursive routine or the calls to nested subroutines are improperly specified, if they are self-referent in a way he does not intend, his program may put the computer into an infinite regress that can be halted only by human intervention. It would be a great help, therefore, if only a general "checking program" could be devised that could inspect any program in a given computer language and determine whether it will always terminate. Unfortunately, such a checking program can be shown to be not just impractical but impossible, and the explanation for the impossibility is to be found in still another insight of mathematical logic, Gödel's theorem, which says that for any axiomatic system that permits self-reference, there will be well-formed, true statements that cannot be derived from the axioms (e.g., Epimenidean self-referent statements that can be paraphrased, "This statement is not a theorem of the system").

Douglas R. Hofstadter is a computer scientist who has clearly thought a great deal about these rather difficult mathematical and logical ideas. For him, they are the key to the understanding not just of computer programs, but of language, art, and the mind itself, and he is anxious to communicate this view to a general audience. "In a way," he says, "this book is a statement of my religion. I hope that this will come through to my readers and that my enthusiasm and reverence for certain ideas will infiltrate the hearts and minds of a few people" [p. xxii].

In order to explain these ideas and the applications he wishes to make, he has adopted a rather unorthodox pedagogical strategy. Instead of simply developing his argument step by step, he shifts back and forth from one theme to another, expounding the same idea repeatedly at increasing levels of complexity. This organization is, however, not haphazard; it is a deliberate attempt to parallel musical form, in particular the form of Bach's A Musical Offering, the fugues and canons in which are supposed to illustrate the very formal structures in which Hofstadter is interested. The logical ideas are presented in a number of expository chapters that include extensive analogies from music and graphic art (this is where Bach and Escher come in), as well as from the natural sciences and Zen Buddhism. The style is clear but hardly elegant ("Gödel realized that there was more here than meets the eye" [p. 18]). Interspersed with the expository chapters are a number of whimsical dialogues, supposed to be parallel in form to various compositions of Bach, in which Achilles hashes over these same ideas with various talking animals,
after the manner of Lewis Carroll (one of the dialogues is in fact a reprint of Carroll's "What the Tortoise said to Achilles").

Hofstadter's pedagogical strategy has the undoubted advantage that even the most abstruse concepts begin to sink in on the nth iteration. Its disadvantage is that, as each new topic is introduced, the reader must have faith that it will eventually prove to be relevant to the main argument. Unfortunately, Hofstadter does many things to weaken one's faith. He is given to quoting long passages needlessly. He includes a great deal of extraneous historical detail (much of it fascinating enough if one hasn't heard it before) about Bach, Babbage, Gödel, Turing, Fermat, Cantor, and others. He decorates the book with portraits of these figures (including one of Turing in athletic costume), as well as hundreds of other unnecessary illustrations. He continually offers such pointless observations as "It is interesting to note that the lives of Mumon and Fibonacci coincided almost exactly: Mumon living from 1183 to 1260 in China, Fibonacci from 1180 to 1250 in Italy" [p. 246]. He cracks a lot of rather poor jokes (one of the dialogues is entitled "SHRDLU toy of man's designing") and then makes matters worse by explaining them. This kind of thing obscures the argument, mars the structure of the book, and makes it much longer (777 pages) than necessary.

In spite of these excesses, it must be said that Hofstadter's "enthusiasm and reverence" for the ideas that fascinate him certainly do come through on every page. And as long as he is explaining the ideas themselves, or relating them to computer programming, he is powerful, lucid, and persuasive. The reader who has found conventional textbook presentations of the ideas of Gödel and Turing difficult to penetrate may well be beguiled into understanding by Hofstadter, and even the reader who already has some glimmerings may find his appreciation of these ideas considerably deepened.

But when Hofstadter tries to demonstrate their pervasiveness in other areas he is not so convincing. As his title suggests, he feels that Bach and Escher, because of their use of recursive structure and self-reference, have some affinity with Gödel. The relevance of these concepts to Escher's drawings is obvious enough: in "Ascending and Descending," for example, a column of monks plods up a stairway whose top somehow appears to be its bottom, and in "Drawing Hands," two hands appear to be drawing each other (these two drawings by Escher, and no less than 32 others, are included among the illustrations). The argument with respect to Bach seems rather less cogent. Though Bach has many ways of varying a theme, and frequently modulates from key to key in a systematic pattern, Hofstadter can really offer only two convincing examples of nested or recursive structure: the "Canon per tonos" in A Musical Offering and the Little Harmonic Labyrinth. The example of "self-reference": the use of the notes B, A, C, H in the incomplete Art of the Fugue, is foolish and irrelevant, and leads only to the bizarre suggestion that Bach fell ill and died before completing this work because of his "attainment of self-reference" [p. 86]. Hofstadter's fascination with Bach's structural devices is as manifest as his fascination with the properties of formal systems, but his "braiding" of the two together often leaves one confused rather than convinced.

Hofstadter is on much firmer ground when he considers the structure of human language. A strong case can be indeed made for including recursive
rules in grammars. As Noam Chomsky has argued, only in this way can one account for the ability of a finite grammar to generate an infinite number of sentences. However, a theory of grammar that simply allowed the unrestricted use of recursive devices would be too powerful: It would permit not only grammars that can occur in natural languages but also an infinite number that cannot. This is the objection to the theory of grammar implicit in an "Augmented Transition Network," a type of recursive procedure which has been used with considerable success by Terry Winograd and others in computer programs for parsing English sentences, and which Hofstadter takes seriously as a model of human sentence parsing. The real problem of the linguistic theoretician is to constrain a grammatical theory permitting recursive devices so that it permits just those grammars that can occur. Hofstadter does not appreciate this point, perhaps because he is, it would appear, aware of current linguistic theory only at second hand: He does not even mention Chomsky.

Having made forays into crystallography and nuclear biology, Hofstadter turns to the problem of modeling human intelligence. Mathematics can not be done except by computation, he argues; since a human mind can solve mathematical problems, its machinery must include some general recursive function for sorting numbers into two classes (this is the "Church-Turing thesis"). What is true of this presumably clear case of human intelligence in action must also be true of other, less well-defined cases. If so, given a non-trivial computer language, it should be possible to write computer programs that simulate other mental activities, and these programs, if successful, must be viewed as veridical models. Such programs, in fact, form the agenda of those computer scientists (of whom Hofstadter is one), who are practitioners of "artificial intelligence." As is well-known, programs have been written that, with varying degrees of success, play chess and checkers, recognize visual and acoustic patterns, synthesize speech, and parse sentences. Eventually, it is suggested, the human mind will be modeled as one large but coherent computer program.

The objection to this argument is not that the Church-Turing thesis is false, but that the extremely modest nature of the psychological claim it makes is disguised. There are uncountable different ways, all compatible with the Church-Turing thesis, in which a human being might conceivably go about solving any particular class of problems, so that a program that models one of these ways is not necessarily of any psychological interest. The mere fact that the program successfully solves the problems set for it, though it may be an impressive demonstration of the programmer's ingenuity, is far from being psychologically conclusive. Indeed, the remarkable success of Arthur Samuels' checker-playing program arouses the suspicion that the specific strategies it uses are quite different from those used by a human player. If so, the program may be telling one a great deal about checkers but not very much about the human mind. Whether the problem is checker-playing or sentence-parsing, the objective should be the development, not of a merely successful program, but of a program that is constrained by what is known of the strategies, effective or not so effective, that human beings actually use. As much recent research in psycholinguistics demonstrates, these strategies can indeed be studied and described.
Is Hofstadter really saying anything save that science is logical? A rigorous model of any natural process must in principle be expressible in a formal system. As he himself makes clear, all but the most trivial of formal systems must allow self-reference and recursive devices, and hence must be subject to the logical limitations expressed by Gödel's theorem. If the model is to make any interesting empirical claims, therefore, it must propose additional constraints of some kind; it is the precise character of these constraints, as has been insisted, that is of primary importance to the physicist, the biologist, or the psychologist. In the absence of such proposals, Hofstadter's arguments come close to being vacuous.