Growth of Surface Waves with Application to the Mucosal Wave of the Vocal Folds

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The initiation of surface waves over a layered structure is discussed. The analysis is done on a plane layered structure consisting of flowing air, epithelium, Reinke’s space, and muscle. Only after the problem is solved for the homogeneous planar structure is a length scale imposed to determine a preferred wavelength, and the conditions for instability derived. Although the layered structure is a highly idealized version of the vocal folds, this analysis illuminates one type of instability allowing for the initiation of the mucosal wave. This same linear analysis can help us understand some of the dynamics of vibration during various parts of the glottal cycle. The method used is similar to that found in the linear analysis of flutter of aircraft panels.

INTRODUCTION

The analysis performed here takes the mucosal surface wave observed during phonation to be a surface wave of a solid, elastic epithelium. The initiation of surface waves on solid or fluid surfaces over which air is flowing is analyzed in many places in the physics and engineering literature. The simplest of this type of analysis is a linear one done on layered structures when the boundaries between the layers are planar. The layers are supposed to be in some initial configuration, for example, with air at some speed flowing over an elastic surface. A time dependent disturbance is added to the operating point, the equations of motion in each layer are linearized with respect to the disturbance, and then solved with boundary conditions between the layers enforced. Because the disturbance is assumed small the linearized equations are supposed adequate to describe the physics well initially. If the disturbance is found to grow in time the system is unstable and a surface wave can be expected to result.

Rayleigh performed linearized planar analyses to study the stability of jets (Rayleigh, 1945). The layers he used were slabs of air moving at different speeds. Lamb, in his monograph on hydrodynamics, shows how linear analysis might apply to the initiation of wind waves over water (Lamb, 1945). Miles has discussed the instability in connection with wind waves over water and oil (Miles, 1959). Engineers have studied linear stability in the context of flutter of aircraft panels during flight (Miles, 1956).

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These latter studies are the most relevant to vocal fold vibration. The analysis here closely parallels the analysis of flutter performed by Miles (1956).

The initiation of the mucosal wave is considered when the vocal folds are assumed to have a planar, layered structure (Figure 1). The layers are based on Hirano's description of the layered structure of the vocal folds. (Hirano 1981, Hirano et al., 1983). The layers going from the midline of the glottis are air, epithelium, Reinke's space, and the deep layer of ligament and muscle. The variations normal to the coronal plane are not considered. Each of these layers has bulk mechanical parameters associated with it, such as density, elasticity, temperature, and so on. It is hoped that from this idealization and with some known values of the mechanical parameters that some aspects of mucosal wave growth can be understood.

![Layered structure of the folds](image)

**Figure 1.** Layered structure of the folds. (The layers are understood to extend along the x-axis to infinity.)

The idea of using a layered structure to model the mechanical properties of the folds is similar to Titze's approach to modeling the vibration of the vocal folds (Titze, 1988; Titze & Strong, 1975; Titze & Talkin, 1979). An important difference is that a complete model along the lines of Titze's is not proposed here. This note is only intended to explore the initiation of the mucosal wave from a layered structure point of view. To this end, the analysis here takes a detailed view of the outer layers, with epithelium and Reinke's space treated as separate layers instead of then being grouped into a single mucosal layer.

Before going on to develop the equations, some limitations of the analysis should be noted. This analysis, initially, does not take account of the inhomogeneity of conditions that really do prevail in each of the layers. For instance, the speed of the air increases substantially through the glottal constriction, which means that there is a natural length scale associated with the flow of the air—the vertical length of maximum glottal constriction. Further, the layers of the folds are inhomogeneous in the vertical direction. Only the portion of the squamous epithelium covering Reinke's space will be considered capable of supporting waves. This suggests the vertical length of Reinke's space treated as separate layers instead of then being grouped into a single mucosal layer.

Before going on to develop the equations, some limitations of the analysis should be noted. This analysis, initially, does not take account of the inhomogeneity of conditions that really do prevail in each of the layers. For instance, the speed of the air increases substantially through the glottal constriction, which means that there is a natural length scale associated with the flow of the air—the vertical length of maximum glottal constriction. Further, the layers of the folds are inhomogeneous in the vertical direction. Only the portion of the squamous epithelium covering Reinke's space will be considered capable of supporting waves. This suggests the vertical length of Reinke's space treated as separate layers instead of then being grouped into a single mucosal layer. We will assume that the maximum glottal constriction length is shorter than the length of Reinke's space. It will be seen that the greater the air...
flow and the longer the surface wavelength, the more likely that instability results. (The shorter wavelength disturbances are more stable because the bending stiffness increases as the fourth power of wavenumber.) As a result, a surface wave is most likely to appear in the glottal constriction with a fundamental wavelength equal to twice the constriction length. (The bulge created in the epithelium will have the glottal constriction length as its length scale, so that the fundamental wavelength of this bulge will be twice the glottal constriction length.) So, while the analysis is initially done on an infinite layered structure, a length scale is imposed later to choose a wavelength to calculate stability regimes. Although not completely consistent, this type of analysis is common and does give us insight to the physical mechanisms responsible for surface wave initiation. Also, the viscous shear stress is neglected in this analysis.

The analysis performed here is a linear analysis, where the equations of motion have been linearized about some assumed initial configuration. If we are willing to make the quasi-steady approximation, the same analysis can be used to describe the dynamics of the vocal fold vibration, locally, in different parts of the glottal cycle.

**ANALYSIS**

A linear stability analysis is applied to the idealized model about an operating point. The operating point to be considered includes uniform airflow at speed \( U \), much less than the speed of sound, \( a_0 \). The density of air is \( \rho_a \) and the viscosity of the air is neglected. The epithelium is supposed to be under tension \( T \) per unit breadth (dimension into the paper in Figure 1), and have bending stiffness, \( D \), per unit breadth. The epithelium is presumed thin compared to any disturbance wavelength, and it has mass per unit area, \( m \). Reinke's space consists of an incompressible fluid of density, \( \rho \), and viscosity is neglected here. The muscle is immovable. This later condition can be made a little more realistic with a general impedance boundary condition. Given this operating point, the linear equations of motion for a small disturbance can be written.

With the neglect of viscosity, the velocity field of a small disturbance in the air can be described with a velocity potential, \( \phi_a \), that satisfies the convected wave equation.

\[
\left( \frac{1}{a_0^2} \frac{D_0^2}{D^2} - \nabla^2 \right) \phi_a = 0,
\]

where

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + U \frac{\partial}{\partial x}.
\]

If \( \zeta \) denotes the inward deflection of the epithelium from rest position \( z = 0 \), (position when there is no pressure difference across the epithelium), then the equation of motion is,

\[
\frac{\partial^2 \zeta}{\partial t^2} + \frac{\partial^4 \zeta}{\partial x^4} - \frac{\partial^2 \zeta}{\partial x^2} = -\Delta p,
\]

where

\[
\Delta p = p_{\|_{x=0}} - p_{\|_{x=0}},
\]

\( p_{\|} \) = pressure of the air,

\( p \) = pressure in Reinke's space.
Because the fluid of Reinke's space is assumed to be inviscid and incompressible, small disturbances in this layer can be described by a velocity potential satisfying,

$$\nabla^2 \phi = 0 \tag{3}$$

The boundary conditions help to determine how the disturbances in each layer interact with one another. Because there is no flow across the midline of the channel, $z = h_0$,

$$\frac{\partial \phi_a}{\partial z} \bigg|_{z=h_0} = 0 \tag{4}$$

Similarly, the muscle is assumed to be immovable, so,

$$\frac{\partial \phi}{\partial z} \bigg|_{z=h_m} = 0 \tag{5}$$

At the epithelium, the velocity at the surface is continuous,

$$\frac{D \xi}{Dt} = \frac{\partial \phi_a}{\partial z} \bigg|_{z=0} \tag{6}$$

and

$$\frac{\partial \zeta}{\partial t} = \frac{\partial \phi}{\partial z} \bigg|_{z=0} \tag{7}$$

The smallness of the disturbance in comparison with the wavelength of the disturbance was used to linearize these boundary conditions, at the plane of equilibrium $z = 0$. (A Taylor series expansion is used to write the boundary conditions on the physical surface at the $z = 0$ plane. Linearization allows all but the first term of the series to be neglected.)

Finally, the relationships between the pressures and velocity potentials are given by linearized Bernoulli’s relations,

$$p_a = -\rho_a \frac{\partial \phi_a}{\partial t} - \rho_a U \frac{\partial \phi_a}{\partial x} = -\rho_a \frac{Du}{Dt} \phi_a$$

and

$$p = -\rho \frac{\partial \phi}{\partial t}$$

The set of equations (1) through (9) forms a closed set.

To find a solution to the above set of equations, assume,

$$\zeta = \zeta_0 \exp(ik(ct - x)) \tag{10}$$

where $k$ is the wavenumber and $c$ is the phase speed.
Because of the boundary conditions equation (6) and equation (7), the velocity potentials must be of the form,

\[ \phi_a = \phi_a(z) \exp(\text{i}k(ct - x)) \]  
\[ \phi = \phi(z) \exp(\text{i}k(ct - x)) \]

Substituting equation (11) into equation (1) gives,

\[ \frac{d^2 \phi_a}{dz^2} - k^2 \left[ 1 - \frac{(c - U)^2}{a_0^2} \right] \phi_a = 0 \]

The solution is,

\[ \tilde{\phi}_a = A' e^{\beta k z} + B' e^{-\beta k z} \]

where

\[ \beta^2 = 1 - \frac{(c - U)^2}{a_0^2} \]

and the constants \( A' \) and \( B' \) are determined by the boundary conditions. Applying equation (4) to the solution equation (14) gives,

\[ \tilde{\phi}_a = A \left( e^{2\beta k z} \right) + B \left( e^{-k z} \right) \]

In a similar way, substituting equation (12) into equation (3), and using the boundary condition expressed in equation (5) gives,

\[ \tilde{\phi} = B \left( e^{2\beta k z} \right) + e^{-k z} \]

Combining equation (6) with equation (8) and (7) with equation (9) gives,

\[ \frac{\partial \rho_a}{\partial z} \bigg|_{z=0} = -\rho_a \frac{D^2 \zeta}{Dt^2} \]

and

\[ \frac{\partial p}{\partial z} \bigg|_{z=0} = -p \frac{\partial^2 \zeta}{\partial t^2} \]
Impedances at the epithelium can be found by using equations (8), (9), (15), and (16),

$$\left. \frac{p_a}{\partial z} \right|_{z=0} = -1 \frac{\beta k \tanh(\beta k_0)}{\beta k \tanh(\beta k_0)}$$

and

$$\left. \frac{p}{\partial z} \right|_{z=0} = \frac{1}{k \tanh(k_m)}$$

Using equations (17) through (20) a homogeneous differential equation replacing the inhomogeneous equation (2) can be written,

$$\frac{\rho_a}{\beta k \tanh(\beta k_0)} \frac{D^2 \xi}{Dt^2} + \left( m + \frac{\rho}{k \tanh(k_m)} \right) \frac{\partial^2 \xi}{\partial t^2} + \frac{D^4 \xi}{\partial x^4} - \frac{T \partial \xi}{\partial x^2} = 0$$

(21)

Substituting equation (10) into equation (21) gives,

$$\left[ 1 + \frac{\rho}{k m \tanh(k_m)} \right] c^2 + \frac{\rho a}{\beta k m \tanh(\beta k_0)} (c - U)^2 \cdot c_0^2 = 0$$

(22)

where

$$c_0^2 = \left( \frac{D k^2 + T}{m} \right)$$

Let $\mu = 1 + (\rho/km) \tanh(k_m)$ and $\mu_a = (\rho_a/\beta km) \tanh(\beta k_0)$. $\mu$ is, essentially, the ratio of the inertial coefficient of the epithelium plus the inertial coefficient of Reinke's space fluid to the inertial coefficient of the epithelium. $\mu_a$ is, essentially, the ratio of the inertial coefficient of the air to that of the epithelium. Equation (22) can be written,

$$(\mu + \mu_a) c^2 - (2\mu a U) c + \left( \mu a U^2 - c_0^2 \right) = 0$$

(23)

In the following it is assumed that $U/a_0$, $c/a_0 << 1$. The air is considered to be incompressible, both because the Mach number of the mean flow is small, and because the length scale of the glottal constriction is small compared to any acoustic wavelength under consideration. (The length scale of concern is the length of the glottal constriction because this is where the maximum velocity occurs. This will be shown to be the most likely place for an instability.) The assumption of incompressibility means that $\beta$ is approximately equal to one, and that a simple closed form solution for the surface wave speed can be obtained. Otherwise, only approximate solutions are available.

Solving for the phase speed,

$$c = \frac{\mu a U \sqrt{c_0^2(\mu + \mu_a) - \mu a U^2}}{\mu + \mu_a}$$

(24)
$c_0$ is the natural phase speed of surface waves on the epithelium without the interaction of the air and Reinke's space. The larger $c_0$, the greater the tension of the epithelium. The effect of airflow is to decrease the ambient pressure (the Bernoulli effect), and hence counter the tension of the epithelium. As the speed of the air increases, the discriminant in equation (24) becomes negative, the phase speed becomes complex, and there is exponential growth in time.

At the point where the discriminant goes to zero, the critical point, the stiffness of the epithelium is just large enough to balance the inward force provided by the air. The air speed at the critical point is given by,

$$U_{\text{crit}} = \sqrt{\frac{1}{\mu} + \frac{1}{\mu_a} c_0}$$

(25)

At the critical point the surface wave phase speed is given,

$$c_{\text{crit}} = \frac{1}{1 + \frac{\mu}{\mu_a}} U_{\text{crit}}$$

(26)

The real part of the phase speed can be sketched as a function of fluid speed, $U$ (Figure 2). Below the critical fluid speed there are two real values for the phase speed. These have opposite signs for $U$ small enough, so that there is both a stable upstream and stable downstream surface wave. However, just below the critical point both stable waves are moving downstream, and at the critical point there is only one phase speed.

Figure 2. Stability Diagram.

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It is interesting to note the dependence of the critical point on the geometric parameters and on the wavelength. These quantities appear in the inertial coefficients $\mu$ and $\mu_a$. As $\mu$ and $\mu_a$ increase the critical air speed decreases (see equation 25). Also, $\mu$ and $\mu_a$ are increasing functions of wavelength, so the longer wavelengths require less air speed for instability. Further, as the air channel gets narrower, that is as $k_h$ gets smaller, the critical air speed becomes smaller. A similar effect occurs to a much smaller magnitude with the thinning of Reinke's space. (The magnitude of the effect is much smaller because $\mu$ is so much larger than $\mu_a$, as is shown below.)

Based on some estimates of the parameters appearing in equations (25) and (26), the critical speeds can be estimated. The air speed is large only through the glottal constriction, and because instabilities depend on the air speed being above a particular critical value, the glottal constriction length is the length scale of interest. Further, the longer wavelengths are the most likely ones to be excited, so that a wavelength twice the glottal constriction length is a reasonable estimate of surface wavelength. The rest of the parameters are set as follows:

vertical length of glottal constriction = 1 cm
$\rho = 1 \text{ g/cm}^3$
$\rho_s = 10^{-3} \text{ g/cm}^3$
density of epithelium $= 1 \text{ g/cm}^3$
thickness of epithelium $= 5 \times 10^{-3} \text{ cm}$ (Titze & Durham, 1987)
this implies $m = 5 \times 10^{-3} \text{ g/cm}^2$
$h_m = 5 \times 10^{-2} \text{ cm}$ (Hirano, et al., 1983)
$h_0 = 10^{-1} \text{ cm}$
$T = 5 \times 10^5 \text{ g/sec}^2$ (Titze & Durham, 1987)

The bending stiffness of the epithelium is neglected and the tension is taken to be that measured at 20% strain.

With these parameter settings equation (25) gives $U_{crit} = 2000 \text{ cm/sec}$ and equation (26) gives $c_{crit} = 1 \text{ cm/sec}$. The critical air speed shown above can be attained in the glottis indicating that the type of instability derived here can lend to the initiation of the mucosal wave. However, the surface wave speed at the critical point shown above is a couple of orders of magnitude less than that observed by Baer (1975) (The surface wave speed reported by Baer was about 1 ms.) This indicates that other mechanisms are also contributing to surface wave growth.

**DISCUSSION**

It is interesting to compare this layered structure approach to the familiar two-mass model. The analysis of the two-mass model shows what needs to be included into this layered structure approach to make it a complete model of mucosal wave instability. Also, it will be seen that the analysis given here can provide insight into the mucosal wave itself and the dynamics of the two-mass model.

The layered structure approach is a continuum mechanics approach, while the two-mass model takes a lumped element approach. The latter has the advantage of easily incorporating the changing properties along the vocal-tract axis, although in a discontinuous way. With the layered structure approach, it is less natural to include these inhomogeneities, but it can be done. The premier advantage of the layered structure approach is the straightforward correspondence between the parameters of the actual folds and the structures in the layers.
It has been shown that mechanical loads must be included in any model of vocal fold vibration to account for the energy exchange between the air and the folds (Stevens, 1977). These loads are necessary to provide for differences between the closing movement and opening movement of the folds in the relation between pressure and surface position. This accounts for the net energy input to the folds from the air. The mechanical loads can be inertial loads of the vocal-tract air or the nonlinear resistances resulting in pressure head losses at the entrance and exit of the glottal regions (Titze, 1988). The two-mass model incorporates both loading effects. A layered structure approach can incorporate both effects also (Titze & Talkin, 1979). Therefore, a layered structure approach in the study of mucosal wave initiation can incorporate these loading effects.

After filling in the noted deficiencies, the resulting layered structure model would be a more complete linear picture of the initiation of the mucosal wave. In fact, the linear instability derived above is not necessary for the initiation of the mucosal wave. All that is needed for linear instability is that there be a negative damping coefficient, but positive restoring force during some part of the glottal cycle. (This instability is analogous to what may be called a dynamic instability in a mass-spring system with negative damping and positive spring constant (Miles, 1959).) In this paper a more catastrophic type of instability has been derived, where the effective restoring force of the epithelium and air system is negative. (This is analogous to what may be called a static instability in a mass-spring system with negative spring constant (Miles, 1959).) In the first case of the dynamic instability, exponentially growing oscillation are the result, and in the latter case of static instability, exponential growth without oscillation results. Titze has shown how the dynamic instability works on a layered structure picture of the vocal folds without "...the underlying mechanisms of surface-wave propagation." (Titze, 1988, p 1540). These mechanisms have been considered here, but with the more severe static instability caused by the Bernoulli pressure overcoming the epithelium tension.

The static instability has not been considered to be important in the analysis of models in the past. Ishizaka & Matsudaira (1972) could have found an instability with an effective negative spring force on the lower mass in their analysis of the two-mass model if they had used a slightly greater subglottal pressure, slightly more compliant springs, and a smaller rest area then the one they used. Titze (1988) recognizes the possibility of such an instability, but removes its consideration from linear problems. It has been shown here that static instabilities can be considered in a linear analysis, where, in fluid mechanics, it is known as a Kelvin-Helmholtz instability (Chandrasekhar, 1981).

This linear analysis can be applied to the parts of the glottal cycle making the quasi-steady approximation, and linearizing the equations of motion about the operating point of instantaneous glottal opening and air velocity. The static instability is most likely to appear in such an analysis during the closing phase of the glottal cycle. The severity and duration of the static of instability may be used to distinguish some voice types. For instance, the conditions of relatively loose epithelium and small (but positive) rest area may describe pressed voice. One would expect that the static instability is more severe and takes a greater portion of the closing phase in the pressed voice than in a chest voice. This offers an alternative explanation to Stevens' account of some of the aspects of pressed phonation, where he assumes that the rest area is negative for pressed voice to obtain the abrupt closure (Stevens, 1988).

To conclude, this layered structure approach may find application in other areas of communicative sound production such as syringeal vibration and trills of various articulators in the vocal tract.

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REFERENCES