SKILLED ACTIONS: A TASK DYNAMIC APPROACH

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Abstract. A task dynamic approach to skilled movements of multidegree of freedom effector systems has been developed in which task-specific, relatively autonomous action units are specified within a functionally defined dynamical framework. Qualitative distinctions among tasks (e.g., the body maintaining a steady vertical posture or the hand reaching to a single spatial target versus cyclic vertical hopping or repetitive hand motion between two spatial targets) are captured by corresponding distinctions among dynamical topologies (e.g., point attractor versus limit cycle dynamics) defined at an abstract task space (or work space) level of description. The approach provides a unified account for several signature properties of skilled actions: trajectory shaping (e.g., hands move along approximately straight lines during unperturbed reaches) and immediate compensation (e.g., spontaneous adjustments occur over an entire effector system if a given part is disturbed en route to a goal). Both of these properties are viewed as implicit consequences of a task's underlying dynamics and, importantly, do not require explicit trajectory plans or replanning procedures. Two versions of task dynamics are derived (control law; network coupling) as possible methods of control and coordination in artificial (robotic, prosthetic) systems, and the network coupling version is explored as a biologically relevant control scheme.

I) Introduction

For animals to function effectively in their environments, their movements must be coordinated in space and time. Though self-evident, this fact

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Acknowledgment. The preparation of this manuscript was supported in part by Contract No. N00014-83-C-0083 from the U.S. Office of Naval Research, NIH grant NS-13617, and Biomedical Research Support Grant RR-05596. Various aspects of the paper have been formally presented at the Second International Conference on Event Perception, Nashville, Tennessee, June, 1983, and the Engineering Foundation Conference on Biomechanics and Neural Control, Henniker, New Hampshire, July, 1983. We would like to express our appreciation to the following colleagues for their helpful comments on an earlier draft of this paper and for valuable discussions concerning several of the topics therein: James Abbs, John Delatizky, Carol Fowler, Louis Goldstein, Vince Gracco, Neville Hogan, John Hollerbach, Fay Horak, Bruce Kay, Wynne Lee, Rich McGowan, Gin McCollum, Paul Milenkovic, Lewis Nashner, Patrick Nye, Marc Raibert, and Michael Turvey. In addition, we are grateful to Phil Rubin who developed some of the basic software procedures used in the present computer simulations.

[HASKINS LABORATORIES: Status Report on Speech Research SR-76 (1983)]
raises a most fundamental issue that has recently attracted a number of disciplines ranging from neuroscience to robotics and cognitive science, viz. how coordination and control arise in complex, multivariable systems. How are the many degrees of freedom adaptively harnessed during coordinated, skilled actions? A deterrent to viable solutions to this problem rests in part in our "limited ability to recognize the significant informational units of movement" (Greene, 1971, p. xviii; see also Szentagothai & Arbib, 1974). For some time, it has seemed questionable to us that nervous systems work through individualized control of component elements, whether they be thought of as joints or muscles. Instead, we believe (and there is an increasing amount of evidence to support the claim) that the many potentially free variables are partitioned naturally into collective functional units within which the component elements may vary relatedly and autonomously. The behavior of these action units or coordinative structures (Fowler, 1977; Kelso, Southard, & Goodman, 1979; Turvey, Shaw, & Mace, 1978) is often exemplified by the existence of relational invariances among kinematic and muscular events during activities as diverse as locomotion, speech, handwriting, and reaching to a target (see Grillner, 1982; Kelso, 1981; Kelso, Tuller, & Harris, 1983; Schmidt, 1982; Viviani & Terzuolo, 1980).

The primary focus of the present paper is to characterize the style of operation of these proposed action units within what we call a task dynamic approach. The term task dynamics follows directly from the view (1) that the degrees of freedom comprising action units are constrained by the particular tasks that animals perform, and (2) that action units are specified in the language of dynamics, not, as is more frequently assumed, in terms of kinematic or muscular variables (cf. Stein, 1982, for an inventory). Thus we propose, and seek to elaborate here, an invariant control structure that is specified dynamically according to task requirements and that gives rise to diverse kinematic consequences.

The paper is organized as follows: First we expand upon those desirable properties of action units that are central to the explication of a task dynamic framework. Second, we present a short tutorial on topological dynamics, a crucial aspect of which is to link the system's geometrical qualities to its dynamics in ways that are task-specific. These steps are precursory to the introduction of the task dynamic approach, two versions of which (control law, network coupling) will be presented. The task dynamic approach will be shown to provide a viable account of such tasks as discrete reaching, bringing a cup to the mouth and turning a handle. It can also offer a principled account of various compensatory behaviors such as those that occur when an arm is perturbed during a reaching movement or when the support base is perturbed during standing. Finally, it will be suggested that the network coupling version of task dynamics both provides an extension of the control law version and offers a new synthesis of recent physiological findings on the planning and control of arm trajectories.

The significance of the task dynamic approach for a theory of coordination and control is that it offers a unified account of certain phenomena that heretofore have required conceptually distinct treatments in the movement literature. In addition, the implications for design and control of robotic and prosthetic devices will be apparent. In fact, the approach shares some but not all of the features of several current developments in manipulator control (cf. Hogan & Cotter, 1982; Raibert, Brown, Cheponis, Hastings, Shreve, & Wimberly, 1981). But before discussing the task dynamic framework
in detail, we will describe the phenomena that led us, in part, to propose the present theoretical approach. Indeed, it is the existence of these phenomena—trajectory shaping and immediate compensation—that constitute the main empirical results that the task dynamic framework is designed to explain.

The first phenomenon, trajectory shaping, refers to the task-specific motion patterns of the terminal devices or end-effectors of the effector systems associated with various types of skill. For example, it has been observed experimentally that, in reaching tasks involving two joints (shoulder and elbow) and two spatial hand motion dimensions, the hands move in quasi-straight-line spatial trajectories from initial to target positions and display single-peaked tangential velocity curves (e.g., Morasso, 1981). Similarly and more obviously, in cup-to-mouth tasks the grasped cup maintains a spillage-preventing, approximately horizontal orientation en route from table to mouth.

The second phenomenon, immediate compensation, refers to the fact that skilled movements show task-specific flexibility in attaining the task goal. If one part of the system is perturbed, blocked, or damaged, the system is able to compensate (assuming the disturbance is not "too big") by reorganizing the activities of the remaining parts in order to achieve the original goal. Further, such readjustments appear to occur automatically without the need to detect the disturbance explicitly, replan a new movement, and execute the new movement plan. Kelso, Tuller, and Fowler (1982) have demonstrated such behavior in the speech articulators (jaw, upper and lower lip, tongue body) when subjects produced the utterances /baeb/ or /baez/ across a series of trials in which the jaw was occasionally and unpredictably tugged downward while moving upward to the final /b/ or /z/ closure (see also Abbs & Gracco, in press; Folkins & Abbs, 1975). The system's response to the jaw perturbation was measured by observing the motions of the jaw and upper and lower lips as well as the electromyographic (EMG) activities of the orbicularis oris superior (upper lip), orbicularis oris inferior (lower lip), and genioglossus (tongue body) muscles. The investigators found relatively "immediate" task-specific compensation (i.e., 20-30 ms from onset of jaw pull to onset of compensatory response) in remote articulators to jaw perturbation. For /baeb/ (in which final lip closure is crucial) they found increased upper lip activity (motion and EMG) relative to the unperturbed control trials but normal tongue activity; for /baez/ (in which final tongue-palate constriction is important) they found increased tongue activity relative to controls, but normal upper lip motion. The speed of these task-specific patterns indicates that compensation does not occur according to traditionally defined "intentional" reaction time processes, but rather according to an automatic, "reflexive" type of organization. However, such an organization is not defined in a hard-wired input/output manner. Instead, these data imply the existence of a selective pattern of coupling or gating among the component articulators that is specific to the utterance produced. Essentially, then, such compensatory behavior represents the classic phenomenon of motor equivalence (Hebb, 1949; Lashley, 1930) according to which a system will find alternate routes to a given goal if an initially traversed route is unexpectedly blocked.

What type of sensorimotor organization could generate, in a task-specific manner, both characteristic-trajectory patterns for unperturbed movements and spontaneous, compensatory behaviors for perturbed movements? We believe that a task-dynamic approach provides at least the beginnings of a cohesive answer to this question. Let us examine these issues, then, beginning with an overview of action unit properties.
II) Units of Action

There are three major points to be made concerning our description of action units:

1. Functional definition; Special purpose device. Action units are defined abstractly in a functional, task-specific fashion and span an ensemble of many muscles or joints. Thus, they are not defined in a traditional reductionist sense relative to single muscles and/or joints, nor are they hard-wired input-output reflex arrangements. These units serve to constrain the muscle/joint components of the collective to act cooperatively in a manner specific to the task at hand. For different skilled actions, performers transform the limbs temporarily into different special purpose devices whose functions match the tasks being performed. Thus, an arm can become a retriever, puncher, or polisher; a leg may become a walker or kicker; the body can become a dancer or swimmer; the speech organs may become talkers, singers, chewers, or swallowers, etc.

2. Autonomy. Action units operate relatively autonomously and are to a large extent self-regulating. That is, once a given functional organization is established over a muscle/joint collective, the system achieves its goal with minimal "voluntary" intervention. In later discussions of the mathematics of task dynamics, we will also indicate that action units are relatively autonomous in a strict mathematical sense, i.e., the equations describing task-dynamic systems are not explicit functions of an independent time variable.

3. Dynamics. Action units are defined in the language of dynamics, not kinematics (e.g., Fowler, Rubin, Remes, & Turvey, 1980; Kelso, Holt, Kugler, & Turvey, 1980; Kugler, Kelso, & Turvey, 1980). The behavior of an effector system is controlled by a task-specific patterning of the system's dynamic parameters (e.g., stiffness, damping, etc.) according to the abstract functional demands of the performed skill. Such dynamical patterning serves to convert the effector system into the appropriate task demanded special purpose device. Further, this patterning both generates the observable motions that are characteristic of that skill and underlies the ability to compensate spontaneously for unpredictable disturbances. There is no explicit plan for the desired kinematic trajectory in the action unit, nor is there an explicit contingency table of replanning procedures for dealing with unexpected perturbations. Rather, task-specific kinematic trajectories and compensatory behaviors emerge from, or are implicit consequences of, the action unit's dynamics. In this sense, most robots (with at least one notable exception, i.e., Raibert et al., 1981) have no skills, but are controlled instead as general-purpose devices using the same dynamical structure for all types of tasks, e.g., spatial trajectory planning for the terminal device, conversion to a joint velocity plan, and joint velocity servoing for both manipulators (e.g., Whitney, 1972) and hexapod walker legs (e.g., McGhee & Iswandhi, 1979).

Given the above three points, one can formulate the problem of skill learning as that of designing an action unit or coordinative structure whose underlying dynamics are appropriate to the skill being learned. That is, in acquiring a skill one is establishing a one-to-one correspondence between the functional characteristics of the skill and the dynamical pattern underlying the performance of that skill. This correspondence between dynamics and function is perhaps the key concept underlying the task-dynamic approach. To explore it more fully we will now: a) examine the geometric notion of topology
as it relates to a system's dynamics; and b) describe how functionally specific dynamical topologies can be used to specify task-specific action units or coordinative structures.

III) Topology and Dynamics

Quite (and perhaps too) simply in the context of skilled action, topology refers to the qualitative aspects of a system's dynamics, e.g., whether a system's dynamics generate 1) a discrete motion to a single target or 2) a sustained cyclic motion between two targets. For a one-degree-of-freedom rotational system such as the elbow joint (flexion-extension degree of freedom) the first motion type might correspond to a positioning task with a single joint angle target, while the second might correspond to a reciprocal tapping task between two joint angle targets. What sort of dynamics might underlie these qualitatively different tasks? For the discrete task, several investigators have hypothesized that the system can be modeled as a damped mass-spring system (e.g., Cooke, 1980; Feldman, 1966; Kelso, 1977; Kelso & Holt, 1980; Politi & Bizzi, 1978; Schmidt & McGown, 1980). Such a dynamical system may be described by the following equation of motion:

\[ I \ddot{x} + b \dot{x} + k(x - x_o) = 0, \]  
where

- \( I \) = moment of inertia about the rotation axis;
- \( b \) = damping (friction) coefficient;
- \( k \) = stiffness coefficient;
- \( x_o \) = equilibrium angle;
- \( x, \dot{x}, \ddot{x} \) = angular displacement and its respective first and second time derivatives.

If we assume a set of constant dynamical parameters \( (I, b, k, x_o) \), then the behavior of this system can be characterized by its point stability or equifinality, in that it will come to rest at the specified \( x_o \) "target" despite various initial conditions for \( x \) and \( \dot{x} \) and despite any transient perturbations encountered en route to the target.

The behavior of such systems can be displayed graphically in two different ways. In Figure 1A, the angle of an underdamped mass-spring with constant coefficients is plotted as a function of time for a given set of initial conditions and with no perturbations introduced. Defining the equilibrium or rest angle as aligned with the abscissa, one observes the system's point stability in the progressive decay of the amplitude to the steady state rest angle. In Figure 1B, the same trajectory is represented alternatively in the phase plane for which the abscissa and ordinate correspond to \( x \) and \( \dot{x} \), respectively, and in which the system's \( x_o \) is located at the phase plane origin. In the phase plane, the system's point stability may be observed as the trajectory spirals down to the origin. Theoretically, if one were to plot the phase plane trajectories corresponding to all possible initial conditions, one would fill the plane with qualitatively similar decaying trajectories defining, thereby, the system's phase portrait. The qualitative "shape" of the system's phase portrait reflects the system's dynamical topology, i.e., the characteristic relations among the system's underlying dynamic parameters. For the type of system described by equation 1, the corresponding phase portrait represents the topology of a point attractor (Abraham & Shaw, 1982), and the
Figure 1. Point attractor system. A. position vs. time; B. Velocity vs. position (phase portrait).

Figure 2. Phase portraits for neutrally stable system (A) and periodic attractor system (B), showing system trajectories for several initial conditions.
underlying dynamics may be described as point attractor dynamics. As a model of discrete positioning tasks, such point attractor dynamics are appealing since the same underlying topology will accommodate different trajectory characteristics (e.g., peak velocity, movement time) and target positions by specification of different values for the system's dynamic parameters.

Obviously, another type of dynamics is required to generate the kinematics observed in a sustained cyclic elbow (or finger, e.g., Kelso, Holt, Rubin, & Kugler, 1981) rotation between two target angles. Perhaps the simplest dynamic scheme corresponds to that of an undamped mass-spring system or harmonic oscillator, with the following equation of motion:

\[ \ddot{x} + k(x - x_0) = 0, \quad (2) \]

where all symbols are defined as in equation (1). The solid line trajectory in Figure 2A represents the phase plane orbit of such a system, which oscillates about the origin \((x_0)\) with an amplitude that is determined by the system's total mechanical energy, and whose angular targets correspond to the system's maximum and minimum angular limits. However, this type of system does not provide a satisfactory model for the cyclic elbow task for two reasons: a) it represents the ideal frictionless case and no real world system is frictionless. Adding friction to equation (2) would simply convert it to equation (1), leaving a point-attractor dynamics unsuitable for any sustained cyclic task; and b) the system described by equation 2 is only neutrally stable in that the oscillation amplitude is extremely plastic with respect to both initial system energy (determined by initial conditions of position and velocity) and transient changes (perturbations) in energy imposed during oscillations. For example, the dotted trajectories in Figure 2A represent oscillations of the same system as does the solid trajectory. However, the inner and outer dotted orbits show the oscillations corresponding to smaller and larger amplitude initial conditions, respectively, relative to the solid orbit. Clearly, then, for a task whose oscillation amplitude is crucial, a neutrally stable system is undesirable.

One can overcome the above shortcomings of an undamped mass-spring dynamics, however, by moving to an alternate periodic attractor (Abraham & Shaw, 1982) dynamical model, with the following equation of motion:

\[ \ddot{x} + b\dot{x} + k(x - x_0) = f(x,\dot{x}), \quad (3) \]

\(I, b, k, x_0, x, \dot{x}, \ddot{x}\) are as in equations 1 and 2; and \(f(x,\dot{x}) = \) nonlinear escapement function of the system's current \(x, \dot{x}\).

This system's behavior is characterized by the three phase plane trajectories seen in Figure 2B corresponding to three different sets of initial conditions. The solid trajectory represents a motion starting at either target, and the inner and outer dotted trajectories represent motions starting inside and outside, respectively, of the target-to-target angular range. It can be seen that these trajectories converge onto the solid orbit, which is described as a stable limit cycle or periodic attractor. In fact, all trajectories (except those starting exactly at \(x_0\)) converge to the limit cycle, and the corresponding phase portrait captures the topology of this periodic attractor.
dynamical system. The reason for this orbital stability lies in the nature of 
the nonlinear escapement term, \( f(x,\dot{x}) \), seen in equation 3.2. Basically this 
term is the means by which the system taps an external energy source in a 
self-gated manner, i.e., energy is gated in or out of the system as a function 
of the system's current \( x,\dot{x} \) state. On the limit cycle, the energy tapped per 
cycle from the external reservoir is equal to the energy dissipated per cycle 
both by the system's intrinsic damping properties (i.e., \( b_2 \)) and the system's 
escapement function. Inside the limit cycle, the energy tapped per cycle is 
greater than that dissipated, and trajectories grow or spiral out to the limit 
cycle; outside the limit cycle, the converse situation holds, and trajectories 
decay or spiral down to the limit cycle (cf. Minorsky, 1962).

The above examples illustrate how particular distinct task functions 
(discrete positioning vs. cyclic alternation) may be modeled by topologically 
distinct dynamical systems. It should be noted, however, that both tasks and 
dynamics were defined in single degree of freedom systems. In these cases one 
dimensional motions were demanded by the tasks and these task requirements 
were mapped directly onto corresponding dynamical control types at a single 
joint. This style of control, in which task-specific sets of constant dynamic 
parameters are defined with respect to control at single joints or articulator 
degrees of freedom, may be labeled articulator dynamics. Real world tasks 
seldom involve such simple one-to-one mappings of task demands into sets of 
constant articulator dynamic parameters. Consider, for example, the two di-
mensional discrete reaching task discussed earlier in the Introduction involv-
ing two articulator degrees of freedom (shoulder, elbow) and two spatial di-
ensions of terminal device (hand) motion. Extending an articulator dynamics 
approach to this more complex task meets with only limited success, providing 
a reasonable account of final position control but failing to account for the 
observed characteristic quasi-straight line hand trajectory patterns 
(Delatizky, 1982). More specifically, in this two dimensional task the arm is 
effectively nonredundant (e.g., Saltzman, 1979) and, given the anatomical 
limits on joint angular excursion, there is a unique mapping from hand posi-
tion to arm configuration (i.e., the set of shoulder and elbow angles; arm 
posture). Therefore, if one defines constant point attractor dynamics at each 
joint with rest angles corresponding to the target arm configuration (and thus 
target hand position), the hand/arm will exhibit equifinality by attaining the 
desired target position/configuration despite variation in initial posi-
tion/posture and despite transient disturbances encountered en route to the 
target. However, as mentioned above (and to be explained in greater detail 
below), such an articulator dynamics approach fails to account for the charac-
teristic trajectory patterns seen in these reaching tasks, i.e., this approach 
does not "favor straight line movements over other movements" (Hollerbach, 

At this point, then, those committed to a dynamical account of coordinat-
ed movement face a nasty dilemma. The conceptually parsimonious account of 
motor control via articulator dynamics no longer appears valid. That is, the 
elegance of the articulator dynamic account for single degree of freedom tasks 
lay in its use of a set of constant, task specific, articulator–dynamic param-
eters to generate a potentially infinite number of task-appropriate kinematic 
trajectories. The failure of such an approach when extended to trajectory 
shaping in a multidegree-of-freedom task as simple as reaching shows that 
searching for invariant task-specific action units at the level of articulator 
dynamics is likely to be a frustrating and probably pointless endeavor. What 
type of principles or control structures might underlie the trajectory con-
straints on arm motion during reaching tasks? There are (at least) two alternative accounts. The first is simply to abandon the dynamical approach altogether, and invoke explicit kinematic trajectory plans as sources for the characteristic constraints on motion patterns observed in different tasks. Such an approach has been generally adopted in the field of robotics (e.g., Hollerbach, 1982; Saltzman, 1979), and has been described in the following fashion by Hollerbach (1982):

A hierarchal movement plan is developed at three levels of abstraction...The top level is the object level, where a task command, such as 'pick up the cup', is converted into a planned trajectory [italics added] for the hand or for the object held by the hand. At the joint level the object trajectory is converted to co-ordinated control of the multiple joints of the human or robotic arm. At the actuator level the joint movements are converted to appropriate motor or muscle activations.

Alternatively, the second account involves defining dynamical control topologies at a level of task description more abstract than the level of individual joints. This leads us to a task-dynamic account of skilled actions.

IV) Task Dynamics

Previous articulator-dynamic descriptions of skilled movement provided plausible accounts of only a very limited type of data: that obtained in laboratory tasks where uni-dimensional tasks mapped directly onto control at a single joint. For example, a discrete target acquisition task was thought to involve specifying the dynamic rest angle parameter corresponding to the task's target joint angle. However, given the failure of articulator dynamics to account for data observed in more complex multivariable tasks, one begins to suspect that this approach might be inappropriate even as a model for control of single variable tasks. More specifically, one reaches the conclusion that the dynamics underlying control of a single joint task might be defined more abstractly than at the articulator level (or joint level; see Hollerbach's, 1982, quote above).

On the basis of a logical analysis of performances across a set of multivariable real world tasks, two common aspects shared by all tasks become evident: a) tasks are typically defined for the terminal devices associated with task-relevant multidegree-of-freedom effector systems (e.g., the grasped cup and arm-trunk, respectively, for a cup-to-mouth task); and b) tasks typically demand characteristic patterns of motion or force by these terminal devices relative to a set of task-specific spatial axes or degrees of freedom. Thus, a given task type can be associated with a corresponding task-spatial coordinate system (task space) that is defined on the basis of both the terminal devices and the environmental objects or surfaces relevant to the task's performance. In fact, Soechting (1982) has presented evidence from a pointing task involving the elbow joint that implies that the controlled variable is not joint angle per se, but rather the orientation angle of the forearm in a spatial coordinate system defined relative to an environmental reference (e.g., the floor surface, or gravity vector, etc.) or the actor's trunk. This suggests that a task-spatial coordinate system might indeed be the appropriate level at which to characterize a skilled action.
The central tenet of the task-dynamic approach is that a set of constant task-dynamic parameters can be defined for each of a given skill's task-space degrees of freedom, defining, thereby, a one-to-one correspondence between the functional characteristics of the skill and the task-dynamical topology underlying that skill's performance. In other words, skill-invariant action units are defined functionally relative to a given skill's task space and underlying task-spatial dynamics (more simply, task dynamics). Such sets of constant task-dynamical parameters may be used to define changing patterns of articulator-level dynamic parameters (e.g., joint stiffnesses, dampings, rest angles, etc.) according to two related versions (control law and network coupling) of the task dynamic approach. The evolving constraints on articulator dynamics serve to convert a given skill's effector system into an appropriate special purpose device whose individual components (i.e., articulator degrees of freedom) act cooperatively in a manner specific to the task at hand. It should be remembered for purposes of comparison that the articulator-dynamics approach postulated sets of constant dynamic parameters at the individual joint level; a given set would underlie the resultant variety of equifinal kinematic trajectories for a given type of single degree of freedom task. In contrast, the task dynamic approach postulates sets of constant dynamic parameters at the task-spatial level for given types of multivariable tasks. A given set of such parameters would: a) underlie directly an articulator-state dependent patterning of articulator dynamic parameters; and b) underlie ultimately the task-specific trajectory patterns and compensatory behaviors observed during task performances. We will now provide an overview of the specifics of the task-dynamic approach, using a relatively simple arm reaching task for illustrative purposes. A schematic of the approach and the coordinate transformations involved is shown in Figure 3.

A. Task dynamics; Task network

1. Task-space. A task-dynamic approach to a given skill begins with an abstract, functional description of that skill's task space. Such a description has three parts. First, the relevant terminal devices and goal objects or surfaces are defined. Second, an appropriate number of task axes or degrees of freedom are defined relative to the terminal device and goal referents; and finally, an appropriate type of task dynamic topology is defined along each task axis. For a discrete reaching task in two spatial dimensions, the corresponding task space is modeled as a two-dimensional point attractor and is illustrated in Figure 4A. In this figure, the reach target (x) defines the origin of a Cartesian coordinate system. Axis $t_1$ (the "reach axis") is oriented along the line from the target to the initial position of the terminal device (open circle), which is modeled as an abstract point task-mass. Axis $t_2$ is defined orthogonal to $t_1$ and measures deviations of the task mass from the reach axis. The task-mass is allowed to assume any $t_1t_2$ position (filled circle) during task performance, and may be considered an abstract point mass since it is not tied to any particular effector system. The equations of motion corresponding to axes $t_1$ and $t_2$ are as follows:

$$m_{T} \dddot{t}_1 + b_{T} \dot{t}_1 + k_{T} t_1 = 0$$

where,

$$m_{T} \dddot{t}_2 + b_{T} \dot{t}_2 + k_{T} t_2 = 0$$

$m_T$ = task-mass coefficient;
Figure 3. Overview of descriptive levels in task dynamic approach.

Figure 4. A. Discrete reaching (task space); B. System trajectories corresponding to different task axis weightings and initial conditions.
In Figure 4A the corresponding damping and stiffness elements are represented in lumped form by the squiggles in the lines connecting the task mass to axes \( t_1 \) and \( t_2 \). Equation (4) describes a linear, uncoupled set of task-spatial dynamic equations, whose terms are defined in units of force, and whose dynamic parameters are constant. This equation can be represented in matrix form as:

\[
M_{T} \dddot{T} + B_{T} \dot{T} + K_{T} T = 0 , \quad \text{where}
\]

\[
M_{T} = \begin{bmatrix} m_{T} & 0 \\ 0 & m_{T} \end{bmatrix} ; \quad B_{T} = \begin{bmatrix} b_{T1} & 0 \\ 0 & b_{T2} \end{bmatrix} ;
\]

\[
K_{T} = \begin{bmatrix} k_{T1} & 0 \\ 0 & k_{T2} \end{bmatrix}
\]

It should be noted that there are two nested structures of dynamical constraints at the task space level. The first constraint structure is defined globally, and serves to establish a task-specific dynamical topology. In our reaching example, these global constraints on the task-dynamic coefficients specified point attractor topologies along each task axis. Additionally, however, a set of locally defined, metrical constraints serve to tune the task-spatial dynamic parameters \( M_{T}, B_{T}, K_{T} \) according to current task demands. Thus, in the reaching example \( m_{T} \) designates the perceptually estimated mass of the terminal device (i.e., gripper + any grasped object-to-be-moved), and \( B_{T} \) and \( K_{T} \) are specified, for example, according to the desired or required damping ratios \( \zeta_{Ti} = b_{Ti}/[2 \sqrt{m_{T}k_{Ti}}] ; i = 1,2 \) and settling times \( T_{Si} = 4/\sqrt{\zeta_{Ti}k_{Ti}/m_{T}} ; i = 1,2, \) i.e., the time required for the system to settle within \( \pm \frac{2}{\zeta_{Ti}} \) of the target amplitude; Dorf, 1974 \) along each task axis.

The movements of the task mass in reaching space display two properties highly desirable for the terminal devices of real world reaching tasks. Due to the point attractor dynamics, the movements will exhibit equifinality in that the task mass will come to rest at the target regardless of initial position (i.e., by definition, initial distance along \( t_1 \) and velocity (i.e., initial direction and speed of task space motion) and despite transient perturbations introduced en route to the target. Additionally, the task mass will show straight line trajectories during unperturbed motions to the target, since in this case the system is effectively one-dimensional by virtue of the definition of the reach axis. However, motions in which the task mass is perturbed away from the reach axis will display trajectory shapes that depend on the relative values of \( k_{T1} \) and \( k_{T2} \) (assuming equivalent damping properties along each axis) as well as the position in \( t_1, t_2 \) space where the perturbation "deposits" the task mass (see Figure 4B). Assuming critical damping along both task axes (i.e., \( \zeta_{Ti} = 1.0 ; \ i = 1,2 \) and a post-perturbation velocity of zero, then: a) when \( k_{T1} < k_{T2} \), the task mass will approach the reach axis faster than axis \( t_2 \); b) when \( k_{T1} > k_{T2} \), the task mass will approach axis \( t_2 \) faster than \( t_1 \); and c) when \( k_{T1} = k_{T2} \), the task
mass will approach \( t_1 \) and \( t_2 \) at the same speed, showing a straight line post-perturbation trajectory to the target. A straight line post-perturbation trajectory will also result if, regardless of the relative values of \( k_{T1} \) and \( k_{T2} \), the task-mass is deposited precisely on the \( t_2 \) axis (Figure 4B, trajectory d). The reason for these relationships between perturbed position, relative axis stiffness, and trajectory shape lies in the shape of the potential energy functions corresponding to these different relative \( k_{T1} \) and \( k_{T2} \) values, and the resultant constraints placed on the ensuing motions of the task-mass when starting at various post-perturbation locations on these manifolds (see Hogan, 1980, for a more detailed discussion of potential energy functions and spring stiffnesses in a similar two-dimensional mass-spring system). Finally, note that these free and perturbed trajectories evolve as implicit consequences of the underlying task space dynamics and, therefore, do not reflect the use of either explicit trajectory plans or replanning procedures.

![Diagram](image)

Figure 5. Discrete reaching: A. Body space. Task space is embedded in a shoulder-centered coordinate system; B. Task network. Body space description is transformed into joint variable form of massless model arm.

2. **Body Space.** The above patterns of task spatial dynamic parameters were defined relative to an environmentally defined goal location and an abstract disembodied terminal device. If these patterns are to be useful to a performer, they must first be transformed into egocentric or body spatial form (e.g., Saltzman, 1979). Such a transformation must be sensitive to the current spatial or geometric relationship between the performer and the task space. As illustrated in Figure 5A for a reaching task, this corresponds to locating and orienting the task space relative to a body spatial \((x_1, x_2)\) coordinate system whose origin corresponds to the current location of the
shoulder's rotation axis. Thus, the terminal device's (task-mass's) current location may be specified in \(x_1x_2\) coordinates. Further, the set of locally defined constraints given by the spatial relationship between task and body spaces serve to tune the body spatial dynamic parameters \(x_0 = (x_{01}, x_{02})^T\) (the location of the task space origin in body space coordinates) and \(\theta\) (the orientation angle between the task space's reach axis \(t_1\) and body space axis \(x_1\)). Given this information, the task-spatial dynamical pattern may be transformed into a corresponding body or shoulder spatial pattern. The resulting set of linear body-spatial equations of motion for the task's terminal device are defined in matrix form as follows (Note: In these and the following equations, a superscript \(T\) denotes the vector matrix transpose operation):

\[
M_B \ddot{x} + B_B \dot{x} + K_B x = 0, \quad \text{where}
\]

\[
M_B = M_T R, \quad \text{where } M_T = \text{task space mass matrix; and}
\]

\[
R = \text{the rotation transformation matrix with elements } r_{ij} \text{ converting task space variables into body space form;}
\]

\[
R = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\]

\[
B_B = B_T R, \quad \text{where } B_T = \text{task space damping matrix}
\]

\[
K_B = K_T R, \quad \text{where } K_T = \text{task space stiffness matrix}
\]

\[
\dot{x} = x - \dot{x}_0, \quad \text{where } x = (x_1, x_2)^T, \text{ the current body space position vector of terminal device; and}
\]

\[
\dot{x}_0 = (x_{01}, x_{02})^T, \text{ the body space position vector of the task space origin.}
\]

One should note that equation (6), unlike equations (4) and (5), represents a set of (usually) coupled, autonomous body spatial dynamic equations (i.e., the off-diagonal terms are generally non-zero) due to the rotation transformation. However, as in the case of the task-dynamic parameters, the terms of (6) are defined in force units and the resultant set of body spatial dynamic parameters is constant.

3. Joint variables; Task Dynamic Network. The above patterns of body spatial dynamic parameters were defined with reference to motions of an abstract terminal device disembodied from its effector system. These patterns may be further transformed into an equivalent expression based on the joint-variables of a massless "model" effector system. Like the transformation from task-space to body space, this transformation is a strictly kinematic one and involves only the substitution of variables defined in one coordinate system for variables defined in another coordinate system. As illustrated in Figure 5B, this corresponds to expressing body spatial variables \((\chi, \dot{\chi}, \ddot{\chi})\) as functions of an arm model's kinematic variables \((\phi, \dot{\phi}, \ddot{\phi})\), where \(\phi = (\phi_1, \phi_2)^T\), \(\phi_1 = \text{shoulder angle defined relative to axis } x_2, \phi_2 = \text{elbow angle defined relative to the upper arm segment. It should be emphasized that the model arm used for this transformation is defined in
kinematic terms only (i.e., the proximal and distal segments have lengths $l_1$ and $l_2$, respectively, but no masses), and that the arm's proximal (shoulder) and distal (wrist) ends are attached to the body space origin and the terminal device/task mass, respectively. The transformed equation is as follows (see Appendix A for details):

$$M_BJ_B\ddot{\mathbf{q}} + B_BJ_B\dot{\mathbf{q}} + K_B\dot{\mathbf{x}}(\mathbf{q}) = -M_BV\ddot{\mathbf{q}}_p,$$  \hspace{1cm} (7)

where $M_B$, $B_B$, $K_B$ are the same constant matrices used in equation (6);

$$\dot{\mathbf{x}}(\mathbf{q}) = \mathbf{x}(\mathbf{q}) - \mathbf{x}_0,$$ where

$$\mathbf{x}(\mathbf{q}) = (x_1(\mathbf{q}), x_2(\mathbf{q}))^T,$$ the current body space position vector of the terminal device expressed as a function of current joint angles;

$$\mathbf{x}_0 = \text{the same constant vector used in equation (6)};$$

$$J = J(\mathbf{q}),$$ the Jacobian transformation matrix whose elements $J_{ij}$ are partial derivatives $\partial x_i/\partial \theta_j$, evaluated at the current $\mathbf{q}_i$;

$$\dot{\mathbf{q}}_p = (\dot{\theta}_1, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_2)^T,$$ the current joint velocity vector; and

$$V = V(\mathbf{q}),$$ a matrix of coefficients associated with $\ddot{\mathbf{q}}_p$ introduced during the kinematic transformation and evaluated at the current $\mathbf{q}$.

One should note that the matrix products in equation (7) are not constant, but are nonlinearly dependent on the current arm model posture $\mathbf{q}$ via the configuration dependence of the $J(\mathbf{q})$ and $V(\mathbf{q})$ matrices. Further, although equation (7) is expressed in terms of articulator or effector system variables, it is by no means an articulator-dynamic equation. Rather, it is simply the body-spatial dynamic equation (6) rewritten in the articulator-kinematic variables of a massless arm model with no reference to the actual mechanics of a performer's corresponding real arm. Its terms, in fact, are still defined in units of force not torque. Thus, if the initial state $(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ for the arm model in equation (7) specifies an initial body-spatial wrist position and velocity equal to the initial position and velocity for the task-mass in equation (6), the arm model's joints will change (via equation (7)) in such a way that the wrist moves along exactly the same trajectory as would the abstract terminal device (via equation (6)).

Equation (7) may be rewritten in units of angular acceleration:

$$\ddot{\mathbf{q}} + J^{-1}M_B^{-1}B_BJ_B\ddot{\mathbf{q}} + J^{-1}M_B^{-1}K_B\dot{\mathbf{x}}(\mathbf{q}) + J^{-1}V\ddot{\mathbf{q}}_p = \mathbf{0},$$  \hspace{1cm} (8)

For reasons to be elaborated further in the sections to follow, we consider equation (7) to define the task dynamic network (task network) for our reaching task example since, in effect, this equation describes a network of task- and context-specific dynamical relations among the arm model's articulator-kinematic variables. Ultimately, however, a reaching task is performed by
a real arm whose motions and responses to perturbations are shaped according to task-specific, evolving patterns of articulator-dynamic parameters. In the task dynamic approach, constraints are supplied for these articulator dynamics with reference to the task network equation (8).

We will now review the basic articulator-dynamics of a simple two-jointed arm, and then discuss two alternative ways in which equation (8) might be used to constrain these dynamics for a reaching task.

B. Articulator dynamics; Articulator network

For the purpose of simplicity, we will restrict our discussion to a two-joint, two-segment effector system whose segments ("upper arm" and "forearm") have lengths $l_1$ and $l_2$, with masses $m_1$ and $m_2$ uniformly distributed along the respective segment lengths. Assuming frictionless revolute joints ($\Theta_1, \Theta_2$; defined in the same manner as for the model arm) and no gravity, the passive mechanical (no controls) articulator dynamic equations of motion, whose terms are defined in units of torque, are (see Appendix B for details):

$$ M_A \ddot{\Theta} + S_A \dot{\Theta}_p = \Theta, \quad \text{where} \quad (9) $$

$$ M_A = M_A(\Theta), \quad \text{the } 2 \times 2 \text{ acceleration sensitivity matrix associated} $$
$$ \quad \text{with inertial torques, whose elements are functions of the current} $$
$$ \quad \text{linkage configuration, } \Theta. \quad \text{The subscript } "A" \text{ denotes articulator} $$
$$ \quad \text{dynamic elements;} $$

$$ S_A = S_A(\Theta), \quad \text{a } 2 \times 3 \text{ matrix associated with coriolis torques} $$
$$ \quad \text{(related to joint velocity cross products) and centripetal torques} $$
$$ \quad \text{(related to squares of joint velocities), whose elements are functions} $$
$$ \quad \text{of the current linkage configuration, } \Theta. $$

With controls included, this equation becomes:

$$ M_A \ddot{\Theta} + S_A \dot{\Theta}_p + B_A \dot{\Theta} + \gamma_{As} + \gamma_{Aa} = \Theta, \quad \text{where} \quad (10) $$

or

$$ K_A \dot{\Theta} $$

$$ B_A = \text{a } 2 \times 2 \text{ control damping matrix;} $$

$$ \gamma_{As} = \text{a } 2 \times 1 \text{ control spatial-spring torque vector;} $$

$$ K_A = \text{a control } 2 \times 2 \text{ joint-stiffness matrix;} $$

$$ \Delta \Theta = \Theta - \Theta_0, \quad \text{where } \Theta_0 = \text{a } 2 \times 1 \text{ control reference configuration} $$
$$ \quad \text{vector;} \quad \text{and} $$

$$ \gamma_{Aa} = \text{a } 2 \times 1 \text{ control additional torque vector, whose function will} $$
$$ \quad \text{be described more fully in the following section on Control Laws.}
Equation (10) may be rewritten as follows with terms defined in units of angular acceleration:

\[ \ddot{\theta} + M_{A_B}^B \ddot{\theta} + M_{A_B}^A \tau_{A_S} + M_{A_B}^A \ddot{\phi} + M_{K_A}^A \tau_{A_A} = \Omega \quad (11) \]

or

\[ M_{A_B}^A \dot{\tau}_{A_A} \]

Just as we considered equation 8 to define a network of task-dynamical relations over a kinematic arm model, we also consider equation 11 to define an articulator dynamic network (articulator network) of relations among our (simplified) real arm's joint variables.

The task dynamic problem for our reaching example (and other real world tasks as well) may now be posed as the question of how to specify patterns of articulator dynamic controls (Equations 10 and 11) such that the resultant terminal device's free and perturbed kinematics evolve according to constraints embodied in the corresponding task space's topological dynamics (Equation 5). We consider two related methods in the sections below based on alternate versions of equation (11). The first method uses equations 8 and 11 to formulate task-specific equations of constraint or control laws over the articulator dynamic parameters; the second combines the use of control laws with the concept of network coupling between the task (equation 8) and articulator (equation 11) networks. Both methods address the issue of coordination in artificial (robotic, prosthetic) linkage systems. The network coupling method also affords a novel perspective on styles of control in physiological systems. In the following section, the control law approach is described, while the network coupling method will be discussed in a later section on physiological modes of motor control.

C. Method 1: Control laws

This method is conceptually quite simple and is illustrated in Figure 6. First, one assumes that the model arm state \((\theta, \dot{\theta})\) equals the real arm state \((\theta, \dot{\theta})\) and that \(\tau\) and \(\dot{\tau}\) (hence, also, \(\phi\) and \(\dot{\phi}\)) are specified proprioceptively. Second, one uses the following version of equation (11):

\[ \ddot{\theta} + M_{A_B}^B \ddot{\theta} + M_{A_B}^A \tau_{A_S} + M_{A_B}^A \ddot{\phi} + M_{K_A}^A \tau_{A_A} = \Omega \quad (12) \]

Third, by comparing equations 8 and 12, one can see that the real arm (\(\Omega\) variables) will move according to task dynamic requirements (i.e., will move identically to the task network's model arm [\(\tau\) variables]) when the following identities hold:  
a) \(J^{-1}M_B B_J = M_{A_B}^B\);  
b) \(J^{-1}M_B B_{A_S}(\theta) = M_{A_B}^A \tau_{A_S}\);  
and c) \(J^{-1}V_{\theta} = M_{A_B}^A \ddot{\phi} + M_{A_B}^A \tau_{A_A}\). Finally, one uses these identities to define the following nonlinear, state-dependent, articulator dynamic control laws:

\[ B_A = M_{A_B}^B J^{-1} M_B B_J \quad (13a) \]
\[ \tau_{A_S} = M_{A_B}^A J^{-1} M_B A_S(\theta) \quad (13b) \]
\[ \tau_{A_A} = (M_{A_B}^A J^{-1} V_{A_S}) \ddot{\phi} \quad (13c) \]
It should be noted that the articulator dynamic controls in equations 13 are defined by the linkage configuration $$\Phi$$- or state $$\Phi$$- dependent products of: a) $$M_A, S_A, J^1, V_x(\Phi),$$ and $$\dot{\Phi}$$—these are $$\Phi$$- or $$\dot{\Phi}$$-dependent, but task independent; and b) $$M_B, R_B, R_B, and x_0$$—these are constant, but are dependent on both spatial context and task. Finally, one should note that for purposes of simplicity we have assumed that the computations involved in equations 13 occur instantaneously. However, in reality this cannot be the case and hence there must be a delay ($$\Delta t$$) between sensing a given linkage state (at $$t = t_i$$) and the specification of a task- and context-specific set of controls (at $$t = t_i + \Delta t$$). It is possible, therefore, that these controls will be totally inappropriate for the current ($$t = t_i + \Delta t$$) linkage state. There are two main ways to deal with this problem. The first is to minimize $$\Delta t$$ by using a variety of methods: a) table lookup (e.g., Raibert, 1978) for those terms in equations 13 that are independent of the current spatial and task contexts, but can be indexed according to current articulatory state; b) parallel computation procedures, such that all elements in all matrices in (13) are not computed sequentially; c) computation strategies that heuristically omit certain terms in (13) or that capitalize on the repeated use of certain "modular" functions (e.g., Benati, Gaglio, Morasso, Tagliasco, & Zaccaria, 1980) in the component terms in (13); and/or d) using remote sensing (expropriation, e.g., vision) to specify certain kinds of information directly (e.g., hand position $$x$$) rather than indirectly through computations based on proprioceptive feedback (e.g., $$x(\Phi)$$). The second way of reducing the adverse consequences of delays is to use a predictive, "lookahead" type of computation (e.g., Ito, 1982; Pellionisz & Linas, 1979) such that given an estimate of delay $$\Delta t$$, the system might sense a linkage state at $$t = t_i$$, predict the state at $$t = t_i + \Delta t$$, and perform equation 13's computations with reference to this predicted state.

Figure 6. Overview of information flow in control law version of task dynamics.

V) Further Examples

In the preceding sections we described the details of the control law version of the task dynamic approach in the context of a discrete reaching
task's point attractor topology. In the present section, we generalize this approach to other task types as well as to variations on the discrete reach task theme. More specifically we describe how the task dynamic model: a) generates task specific trajectory shapes in discrete reaching, rhythmic target-to-target, cup-to-mouth, and crank-turning tasks; and b) provides "immediate compensation" to a sustained perturbation introduced to an effector system while en route to a target in a reaching task.

In the current control law context, all examples and computer simulations described below represent motions of the articulator network (the "real" arm), and the task network (the "model" arm) is rigidly constrained to move identically due to the assumptions that \( \ddot{\phi} = \ddot{\phi} = \ddot{\phi} = \ddot{\phi} \), given the current "proprioceptively" specified \( \ddot{\phi} \) and \( \ddot{\phi} \).

A. Trajectory Shaping

1. Discrete reaching. This is the familiar reaching example, whose task space is defined as a two-dimensional point attractor (see Figure 4A). A straight-line trajectory for the terminal device (the hand) generated by these task dynamics for a discrete reach is illustrated in Figure 7 (trajectory a). For this trajectory the task space axis stiffnesses are symmetrical (i.e., \( k_{T1} = k_{T2} \)) and critical damping is assumed along both axes. Note, however, that perfect straight line trajectories are generated in contrast to the quasi-straight line trajectories observed experimentally for primates (e.g., Georgopoulos, Kalaska, & Massey, 1981; Morasso, 1981; Soechting & Lacquanti, 1981).

Figure 7. Body space discrete reaching trajectories showing effects of omitting velocity product torque compensation terms with different task axis weightings. I and F denote initial and final arm configurations, respectively.
As alluded to earlier (see also footnote 8), it is possible to omit \( \mathcal{A}_{\text{aa}} \) (i.e., the control vector associated with velocity product torques) from equation 12 and thereby obtain more "realistic" trajectories while at the same time reducing the amount of computation involved in specifying constraints on articulator dynamic parameters (trajectory b in Figure 7). As with trajectory a, trajectory b illustrates a reach involving symmetrical task axis stiffnesses and critical task space damping. Omitting \( \mathcal{A}_{\text{aa}} \) results in an articulator network whose velocity product terms are simply those specified by passive arm mechanics (i.e., \( \mathbf{S}_a \mathbf{Q} \) in equation 9) rather than those specified by the task network (i.e., \( \mathbf{M}_b \mathbf{V}_a \) in equation 7). Note that although the omission of \( \mathcal{A}_{\text{aa}} \) introduces a "hook" into trajectory b's illustrated hand motion, the hand nevertheless arrives precisely on target due to the underlying task space point attractor dynamics. This preservation of accurate targeting behavior when control terms related to velocity product torques are ignored is a feature of the task dynamic approach not shared by some other robotic control schemes (e.g., Hollerbach & Flash, 1981, see their Figure 8). Finally, it should be noted that straight line hand trajectories can be approximated when \( \mathcal{A}_{\text{aa}} \) is omitted by a judicious relative weighting of task axis stiffnesses. Hand trajectories progressively closer to ideal straight lines will be produced using progressively greater penalties for task mass deviations from the taskspace reach axis \( (t_1) \) en route to the target. A hand trajectory for the arm motion corresponding to one such ratio \( (k_2 : k_1 = 1.75 : 1) \) with critical damping along both task axes is illustrated in Figure 7 (trajectory c).

2. Cup-to-mouth task. In a cup-to-mouth task the goal is to move a cup of liquid from an initial to final position (e.g., table top to mouth) while maintaining a horizontal spillage-preventing cup orientation during the movement. As in our discussion of the discrete reaching task, we begin with a simplified task-dynamic treatment of a planar cup-to-mouth task performed by a 3-joint (shoulder, elbow, wrist) arm using an abstract, functional description of that skill's task space. This task space is modeled as a three dimensional (one rotational and two linear degrees of freedom) point attractor and is illustrated in Figure 8A. In this figure the terminal device is an abstract task-segment \( (m_T = \text{mass}, l_T = \text{length}) \) representing the grasped cup, with one end (the "distal" end) defined as the point of final cup-mouth contact, and requiring three coordinates for its complete task space description. The target location (mouth) for the segment's distal end defines the origin \( (t_{01}, t_{02}) \) of a \( t_1 t_2 \) Cartesian coordinate system; axis \( t_1 \) is defined as a reach-axis from the initial position of the segment's distal end to the \( t_1 t_2 \) origin; and axis \( t_2 \) is defined orthogonally to \( t_1 \). The orientation of the task segment relative to axis \( t_1 \) defines the current angular \( t_3 \) coordinate; \( t_{03} \) defines the (identical) initial and target task segment orientations; and \( I_T (= l_1 / 3 m_T l_T) \) is the task segment's moment of inertia about its distal end. The equations of motion corresponding to axes \( t_1 \), \( t_2 \), and \( t_3 \) are:

\[
\begin{align*}
 m_T \ddot{t}_1 + b_T \dot{t}_1 + k_{T1} t_1 &= 0 \quad &\text{(14a)} \\
 m_T \ddot{t}_2 + b_T \dot{t}_2 + k_{T2} t_2 &= 0 \quad &\text{(14b)} \\
 \rho \ddot{t}_3 + \rho b_T \dot{t}_3 + \rho k_{T3} (t_3 - t_{03}) &= 0 \quad &\text{(14c)}
\end{align*}
\]

where \( \rho \) is a constant scaling factor with units of length and is used to ensure dimensional homogeneity along all task space degrees of freedom. Thus,
Figure 8. Cup-to-mouth task: A. Task space; B. Body space; C. Task network.
all terms of equation 14, even the rotational terms of $14c$, are defined in units of force. For purposes of the present paper, $\rho$ is set to 1 and consequently is omitted for notational simplicity in all further discussions in this section. In Figure 8A the stiffness and damping elements are represented in lumped form as squiggles in the lines connecting the task segment to the linear "rest positions" and to the rotational "rest orientation." Equations 14 describe a set of uncoupled (by definition of the abstract task space) equations with constant task dynamic parameters and can be represented in matrix form as:

$$
M_T \ddot{T} + B_T \dot{T} + K_T T = 0 \text{, where}
$$

(15)

$M_T$, $B_T$, and $K_T$ are $3 \times 3$ diagonal matrices of task dynamic parameters analogous to the simpler $2 \times 2$ point attractor system of equation 5. In a similar fashion, the body spatial equation and the joint variable (task network) equation are simply the $3 \times 3$ analogs of equations 6 and 7. The corresponding body spatial and joint variable representations are illustrated in Figures 8B and 8C.

When simulated, a typical movement generated by these task dynamics, using symmetrical task axis stiffnesses ($k_{T1} = k_{T2} = k_{T3}$) and critical damping along all task axes, shows both a straight line trajectory and a maintained horizontal orientation of the task segment during the movement.

3. Reaching (rhythmic). The point attractor task space topologies used for the discrete reaching and cup-to-mouth tasks will be unable to generate the arm kinematics associated with sustained cyclic hand motion between two body spatial targets. Consider, for example, the case of planar motion of the terminal device (hand) and a corresponding 2-joint effector system (arm with shoulder and elbow joints). The task space is illustrated in Figure 9A and consists of an orthogonal pair of axes $(t_1, t_2)$ for which: a) $t_1$ is defined along the line between the two targets ($D$=distance between the targets); and b) the origin is located midway between the two targets ($A=D/2$=distance from origin to either target). The terminal device is an abstract point task-mass ($m_T$=mass), and may be located anywhere in the task space. Point attractor dynamics are defined along axis $t_2$ to bring the task mass onto axis $t_1$ and to maintain it there despite transient perturbations introduced perpendicular to $t_1$. Limit cycle (periodic attractor) dynamics are defined along axis $t_1$ to sustain a cyclic motion of the task mass parallel to $t_1$ between the two targets, and to maintain the desired oscillation amplitude ($A=D/2$) despite perturbations introduced parallel to $t_1$. The task space equations of motion are:

$$
\begin{align*}
\dot{m}_{T1} + b_{T1} \dot{t}_1 + c_{T1} t_1^3 + k_{T1} t_1 &= 0 \\
m_{T2} + b_{T2} \dot{t}_2 + k_{T2} t_2 &= 0
\end{align*}
$$

(16)

$m_T$, $k_{T1}$, $b_{T2}$, and $k_{T1}$ are defined as in equation 5 (discrete reaching task, point attractor); and $(-b_{T1} \dot{t}_1 + c_{T1} t_1^3)$ is the nonlinear escapement term (van der Pol type) for axis $t_1$.

The dynamic parameters for axis $t_2$ are tuned in the same manner as in the $t_2$ axis of the discrete reach task space (see earlier Task space section). Tuning the dynamic parameters along axis $t_1$ involves specifying $k_{T1}$ according to the desired period, $P$, of motion and the relation
Figure 9. Rhythmic reaching: A. Task space. Open circles represent targets. Squiggle represents point attractor (spring and damper) dynamics along axis $t_2$. Open box represents limit cycle (spring and van der Pol escape) dynamics along axis $t_1$; B. Body space; C. Task network.
The procedure for specifying \( b_{T1} \) and \( c_{T1} \) is more involved, and may be understood by considering equation 16a in the following normalized, dimensionless form:

\[
Z_1'' - \varepsilon (1-Z_1^2)Z_1' + Z_1 = 0, \text{ where} \tag{17}
\]

a) the single and double apostrophe superscripts denote differentiation with respect to the dimensionless time variable, \( N = \Omega \cdot n \), with \( \Omega = \sqrt{K_{T1}/m_T} \) and \( n \) denotes the standard time variable; b) \( Z_1 = c_{T1}/b_{T1} \cdot t_1 \) is the dimensionless displacement variable; and c) \( \varepsilon = b_{T1}/m_{T1}^{1/2} \) is a dimensionless measure directly related both to escapement "strength" (i.e., the strength with which the system resists being displaced from the limit cycle) and the shape of the limit cycle orbit in the phase plane (e.g., \( \varepsilon \ll 1 \) corresponds to a circular orbit and sinusoidal motion; \( \varepsilon \gg 1 \) corresponds to a relaxation orbit and a step-like motion). Given values for \( K_{T1} \) (from the desired period) and \( \varepsilon \) (from the desired orbit shape), \( b_{T1} \) is determined from the above expression for \( \varepsilon \). Finally, it is known that the amplitude for the normalized (Z-variable) system of equation 17 is equal to 2.0 over a wide range of \( \varepsilon \) values (e.g., \( 0 \leq \varepsilon \leq 10 \), Jordan & Smith, 1977). For the original non-normalized (x-variable) system of equation 16a, the corresponding amplitude is \( A = 2 \cdot b_{T1}/c_{T1} \). Therefore, given values for \( b_{T1} \) and desired amplitude \( A \), the value of \( c_{T1} \) is determined from the preceding expression for \( A \). Finally, equation 16 may be rewritten in matrix form as:

\[
M_{T}^{\ddot{x}} + B_{T}^{\dot{x}} + K_{T}x = F_{T}, \text{ where} \tag{18}
\]

\( M_{T} \) and \( K_{T} \) are defined as in equation 5;

\[
B_{T} = \begin{bmatrix}
-b_{T1} & 0 \\
0 & b_{T2}
\end{bmatrix}, \text{ denoting the linear damping components; and}
\]

\[
F_{T} = (-c_{T1} \cdot t_1', 0)^{T}, \text{ denoting nonlinear system components.}
\]

Equations 16 and 18 represent an autonomous, uncoupled, task spatial dynamical system with constant parameters. Figure 9B illustrates how the task space is located and oriented in body (shoulder) space. The body spatial dynamical system is described by:

\[
M_{B}^{\ddot{x}} + B_{B}^{\dot{x}} + K_{B}x = F_{B}, \text{ where} \tag{19}
\]

\( M_{B} = M_{T}^{R} \), where \( R \) is the rotational transform matrix with elements \( r_{ij} \) defined previously in equation 6;

\[
B_{B} = B_{T}^{R};
\]

\[
K_{B} = K_{T}^{R}; \text{ and}
\]

\[
F_{B} = (-c_{T1}(r_{11} \dot{x}_1 + r_{12} \dot{x}_2), (r_{11} \dot{x}_1 + r_{12} \dot{x}_2), 0)^{T}.
\]

Equation 19 describes an autonomous, coupled (due to the rotation transformation), body spatial dynamical system with a constant set of linear parameters and a nonlinear, state-dependent forcing function. This body spa-
tial equation may be transformed kinematically into joint variable form by expressing \( x \) variables as functions of the \( \theta \) variables of a corresponding model arm (see Figure 9C):

\[
M_B \ddot{\theta} + B_B J \ddot{\theta} + K_B \Delta x(\theta) = \dot{F}_\theta = M_B \ddot{\theta}_p, \quad \text{where} \tag{20}
\]

\( M_B, B_B, \) and \( K_B \) are defined as in equation 19;

\( J = J(\theta), \) the Jacobian matrix;

\( \Delta x(\theta) = x(\theta) - x_0; \)

\( F_\theta = F_B(\theta), \) i.e., \( F_B \) with the substitutions \( \Delta x_1 = \Delta x_1(\theta), \Delta x_2 = \Delta x_2(\theta), \dot{x}_1 = J_{11} \dot{\theta}_1 + J_{12} \dot{\theta}_2, \dot{x}_2 = J_{21} \dot{\theta}_1 + J_{22} \dot{\theta}_2; \) and

\( V \) and \( \dot{\theta}_p \) are as defined in equation 7.

Equation 20 may be rewritten in the following task network form:

\[
\ddot{\theta} + J^{-1} M_B^{-1} B_B J \ddot{\theta} + J^{-1} M_B^{-1} K_B \Delta x(\theta) = J^{-1} M_B^{-1} F_\theta - J^{-1} V \dot{\theta}_p \tag{21}
\]

Figure 10. Body space rhythmic reaching trajectory when hand starts (or is perturbed to) a position away from the steady state trajectory.

Finally, since the real arm's motion can be described by the articulator network \( (\theta) \) variables) equation 12, one sees that the articulator controls, \( B_A \)
and \( \gamma_{as} \) are specified according to control laws 13a and 13b, respectively. A comparison of equations 12 and 21 shows that, assuming \( \phi = \phi_0 \) and \( \phi = \phi_2 \), the articulator control \( \gamma_{aa} \) is defined according to the following control law:

\[
\gamma_{aa} = (M_A J^{-1} V - S_A) \phi_p^- - M_A J_{B}^{-1} N_{B}^{-1} \phi
\]

(22)

A typical movement generated by these task-dynamics is illustrated in Figure 10, showing the motion of the task mass in body (shoulder) space. Note the straight line hand trajectory during the steady-state cyclic motion between targets, and also the way the hand is attracted autonomously to this steady-state trajectory despite a startup position (with zero velocity) away from this trajectory.

4. Crank Turning. Figure 11C illustrates the shoulder spatial layout of a crank turning task in which: a) motion of arm and crank occur in the horizontal plane; b) the crank segment's "distal" end is attached to a fixed rotation axis located at \( \phi_0 \) in shoulder space; c) the crank rotates at a constant angular velocity, \( V \), about the fixed axis; d) the wrist joint is fixed and the hand tightly grasps the crank's handle, which freely rotates about an axis fixed to the crank's "proximal" end; and e) \( \phi_1 \) and \( \phi_2 \) represent the shoulder and elbow angles, respectively, while \( \phi_3 \) represents the angle between the hand-forearm and crank. The task space description is illustrated in Figure 11A in which: a) the crank is the terminal device or task segment \( (m_t = mass, l_t = length) \); b) the fixed rotation axis at the crank's distal end defines the origin of a Cartesian \( t_1 \) \( t_2 \) coordinate system; c) an angular \( t_3 \) coordinate is defined by the orientation of the crank relative to axis \( t_2 \); and d) \( I_T = (1/3)m_t l_t^2 \) is the crank's moment of inertia about its distal end. The task spatial equations of motion are defined as:

\[
m_{t_1} \ddot{t}_1 + b_{t_1} \dot{t}_1 + k_{t_1} t_1 = 0 \quad (23a)
\]

\[
m_{t_2} \ddot{t}_2 + b_{t_2} \dot{t}_2 + k_{t_2} t_2 = 0 \quad (23b)
\]

\[
\rho I_{t_3} \ddot{t}_3 - \rho b_{t_3} \dot{t}_3 + \rho c_{t_3}^{13} \dot{t}_3 = 0 \quad (23c)
\]

\( \rho \) is the same scaling factor used in equation 14c, and will be omitted from further discussions in this section for notational simplicity.

Equations 23a and 23b define point attractors whose corresponding damping and stiffness factors are represented in lumped form in Figure 13a, and which serve to maintain the crank's distal end at the task space origin. Since in the real world the crank is fixed to this axis, these axes may be weighted rather loosely (i.e., may be assigned low values for \( k_{t_1} \) and \( k_{t_2} \)). Equation 23c needs a bit more explanation as it contains a limit cycle's escapement term (Rayleigh type escapement: \( -b_{t_3} \dot{t}_3 + c_{t_3}^{13} \)) but no spring term. The behavior associated with equation 23c is best understood by examination of its corresponding phase portrait (Figure 12). Here it can be seen that there are three steady states represented by lines parallel to the \( t_3 \) axis. The lines defined by \( \dot{t}_3 = \pm V = \pm b_{t_3} / c_{t_3}^{13} \) are stable steady states, and the line \( t_3 = 0 \) denotes an unstable steady state. In other words, given any nonzero startup velocity in either the upper or lower half plane, the system will reach the corresponding positive or negative steady state angular velocity, \( \dot{V} \). If, however, the system begins at any angular
Figure 11. Crank turning: A. Task space. Squiggles represent point attractor dynamics along linear axes $t_1$ and $t_2$. Open box represents velocity attractor (Rayleigh escapement) dynamics along rotational axis $t_3$; B. Body space; C. Task network.
position with precisely zero velocity, it will simply stay at that position. In normalized form, equation 23c becomes:

\[ \ddot{Z}_3 - \epsilon(1 - \dot{Z}_3^2)\dot{Z}_3 = 0, \]

where

\[ Z_3 = \sqrt{\frac{c_{T3}}{b_{T3}}} t_3 \] is the dimensionless displacement variable, and

\[ \epsilon = \frac{b_{T3}}{l_T} \] is directly related to the "strength" of the escapement (i.e., the speed with which the system attains the steady state and the strength with which it resists perturbations from the steady state). Since we are unaware of any other label for this type of dynamical topology, we will call it a bistable velocity attractor (or more simply, a velocity attractor). Given a desired escapement strength (\( \epsilon \)) and final crank angular velocity (\( V \)), the above relationships are sufficient to tune the system's \( b_{T1} \) and \( c_{T1} \) values according to these task demands. Equations 23 may be rewritten in matrix form as:

\[ M_T \dddot{t} + B_T \ddot{t} + K_T \dot{t} = F_T, \]

where

\[ M_T \] is defined as in equation 15;

\[ B_T = \begin{bmatrix} b_{T1} & 0 & 0 \\ 0 & b_{T2} & 0 \\ 0 & 0 & -b_{T3} \end{bmatrix} \]

\[ K_T = \begin{bmatrix} k_{T1} & 0 & 0 \\ 0 & k_{T2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

and

\[ F_T = \begin{bmatrix} 0 \\ 0 \\ -c_{T3} \dot{t}_3 \end{bmatrix}^T \]
Equations 23 and 25 represent an autonomous, uncoupled (by definition) task spatial dynamical system with constant parameters. Figure 11B shows how the task space is located and oriented in body (shoulder) spatial coordinates. Note that the orientation of task-to-body space is arbitrary, and in Figure 11B the orientation angle $\Omega$ is simply assumed to be zero (e.g., see Figure 8B for an example of a different task with nonzero $\Omega$). Figure 11C, as mentioned previously, shows the relation of the task and body spaces to the task's "model" arm. Equations for body spatial, arm model, and task network dynamics may be derived from equation 25 in a manner similar to that used in generating equations 19, 20, and 21, respectively, from equation 18.

It should be noted that the configuration $\mathbf{q}$ of the model arm is specified in exactly the same manner as our earlier examples. Angles $\phi_1$ (shoulder) and $\phi_2$ (elbow) can be obtained "proprioceptively" but $\phi_3$, the angle between the crank and the hand–forearm, cannot. However, assuming that the location of the crank's distal end (environmentally fixed rotation axis) is known in body space coordinates and given $\phi_1$ and $\phi_2$ proprioceptively, $\phi_3$ is uniquely specified by geometric considerations. Thus the full $\mathbf{q}$ set is available for use in the control law computations.

B. Immediate Compensation

In the Introduction, we reviewed experimental data on speech movements that showed task-specific, automatic, compensatory response patterns in remote articulators to unpredicted transient perturbations in a given articulator that were relatively immediate. These data implied that selective patterns of coupling or gating existed among the component articulators that were specific to the produced utterances. In the context of the task dynamic approach, we hypothesize that these coupling patterns are due to the corresponding evolving patterns of articulator-dynamic control parameters specified by task- and state-dependent control laws or equations of constraint.

To illustrate, consider the following example of a discrete reaching task (formulated as a modified version of a cup-to-mouth task) in which: a) the terminal device is a pointer fixed to the hand of a 3-segment (upper arm, forearm, hand-pointer) arm; b) planar motion of the pointer corresponds to angular motions of the arm's 3 joints ($\phi_1$-shoulder, $\phi_2$-elbow, $\phi_3$-angle between pointer and forearm); and c) task demands focus on positioning the pointer's distal end at a body-spatial $x_1x_2$ target but are relatively indifferent to the precision of final orientation control. Consequently, the task space may be described as a 3-dimensional point attractor with symmetrical weightings for the linear $t_1$ and $t_2$ axes, and a much smaller weighting for the rotational $t_3$ axis. Figure 13 illustrates the initial (a) and final (b) arm configurations that correspond to the current task dynamics (weighting ratio of axes $t_1$ and $t_2$ to $t_3$ is 20:1) when the arm encounters no perturbations en route to its body spatial pointer target. The initial arm configuration is $\mathbf{q}_i = (79^\circ, 20^\circ, 1171^\circ)^T$ and the final arm configuration $\mathbf{q}_f = (115^\circ, 81^\circ, 75^\circ)^T$. Figure 13 (configuration c) shows the final arm position when the shoulder angle is suddenly braked during the trajectory when it reaches $105^\circ$ and is held fixed at this angle. The initial $\mathbf{q}_i$ is the same as in the unperturbed case and the pointer's distal end reaches precisely the same spatial $x_1x_2$ target as in the unperturbed motion, despite the fact that the final configuration has changed to $\mathbf{q}_f = (105^\circ, 95^\circ, 52^\circ)^T$. In other words, the system's response to the perturbation was to "automatically" redistribute the activity among its component degrees of freedom in a
Figure 13. Arm configurations for simulated discrete reaches showing:
a. Initial posture; b. Final posture (unperturbed trajectory); c. Final posture (perturbed trajectory).

Figure 14. Description of basic postural perturbation paradigm of Nashner and colleagues, showing four types of perturbation (right column) and corresponding leg joint angular rotations (left column); A. AP translation; B. Direct rotation; C. Synchronous vertical; D. Reciprocal vertical. (Adapted from Nashner & Woolacott, 1979.)
manner that still achieved the same task spatial goal. Furthermore, such compensatory motor equivalence reflects the fact that targets in the task dy-
namic approach are not specified as final articulator configurations, but rather as desired final spatial coordinates for the terminal device; the fi-
nal articulator configuration "falls out" of the task dynamic organization and the environmental conditions in which the movement is performed.

VI) Relevance to Physiological Literature

In this section, we will describe how the task dynamic approach might ap-
ply to the issue of postural control in humans, and suggest a "systemic" al-
ternative to the modular synergy model of Nashner (e.g., Nashner, 1981;
Nashner & Woolacott, 1979) to account for postural compensatory phenomena. Further, we will review evidence from studies of single-joint discrete move-
ment tasks (e.g., Bizzi, Chapple, & Hogan, 1982) that show that the physiolo-
gically relevant parameter of rest angle (i.e., the angle specified by the equilib-
rium point between agonist and antagonist length-tension curves) is ac-
tively specified during such tasks as a gradually (as opposed to step-like) changing central control signal. In the control law version of task dynamics, there is no articulator-dynamic control parameter corresponding to rest angle. However, a network coupling version of task dynamics, now in preliminary form, will be described that includes rest angle as a parameter and provides a ra-
tional account for the evolution of the rest angle's trajectory without requiring an explicitly preplanned trajectory representation to account for the observed pattern. Finally we will discuss the implications of the network coupling approach for theories of learned complex skilled actions.

A. Postural control

Nashner and his colleagues have performed an elegant series of experi-
ments on postural responses to support surface perturbations in standing human subjects. Summarizing from the experimental report of Nashner, Woolacott, and Tuma (1979) and several subsequent reviews (Nashner, 1979; Nashner, 1981;
Nashner & Woolacott, 1979), we can describe the paradigm and findings in the following way. Basically, a subject stands with each foot on a separate horizontal platform that can be translated horizontally, translated vertically, or rotated about an axis aligned with the ankle joint. Using these platforms, one or a combination of the following four types of perturbation could be delivered to the subjects on a given trial (Figure 14): a) simultaneous forward or backward anteroposterior translation (AP translation); b) simultaneous flexion or extension rotations (direct rotation), c) simultaneous upward or downward vertical translation (synchronous vertical); and d) reciprocal vertical translation (reciprocal vertical). These perturba-
tion types may be characterized by the corresponding patterns of whole body motions and joint rotations that would be induced in "passive" noncompensating subjects (Figure 14). Thus, AP translation caused the body to lean in the direction opposite to the translation; direct rotation caused the body to tilt in the same sense as the rotation; synchronous vertical caused the body to move with the translation; and reciprocal vertical caused the body to tilt laterally toward the lowering platform. It should be noted that the first three perturbation types induce motions in the sagittal plane, while the reciprocal vertical type induces motion in the frontal plane.
In response to each perturbation type or type combination, Nashner et al. measured EMG responses from the upper and lower leg muscles, as well as changes in ankle, knee, and hip angles. Associated with each perturbation type was a long latency (e.g., 100-110 ms latency in gastronemius) "rapid postural adjustment" (Nashner, 1981), which comprised the earliest useful postural response, while the shorter latency myotatic reflexes were either absent or of no apparent functional value. These rapid postural adjustments for a given type a) were characterized by fixed ratios of activity among the responding muscles, b) were specific to the perturbation type (and the corresponding type-specific patterns of joint displacements), and c) were "functionally related to the task of coordinating one kind of postural adjustment" (Nashner, 1979, p. 179). Further, during a set of trials in which a sequence of either of three perturbation types (AP translation, synchronous vertical, or reciprocal vertical) was unexpectedly and immediately followed by a sequence of one of the other two types, it was found that the functionally appropriate postural synergy response occurred even on the first trial of the new type. Such "first trial adaptation" did not occur, however, when a sequence of AP translations was followed immediately by an unexpected series of direct rotations (or vice versa). In these cases, the functionally correct (i.e., posturally stabilizing) synergistic response pattern was implemented progressively over a series of approximately three to five trials. Additionally (Horak & Nashner, 1983), if a series of AP trials with the subject standing directly on the footplates was followed by a series of AP trials with the feet resting on narrow transverse beams, the subjects switched from a postural response involving predominantly ankle motions (ankle strategy) to one involving predominantly hip motions (hip strategy). This strategy change was implemented progressively over the course of approximately 5-20 trials, and this multtrial adaptation process was also seen for the reverse change from beam to footplate postural strategies.

Nashner and his colleagues have interpreted these data as being consistent with a modular synergy "conceptual model for the organization of postural adjustments" (e.g., Nashner, 1979, 1981; Nashner & Woolacott, 1979). Although admittedly in preliminary form, this hierarchical model proposes that postural synergies are organized spinaly as separate modular function generators, and are automatically triggered by correspondingly appropriate distinctive features of somatosensory (i.e., proprioceptive information related to joint angular rotations) inputs. Thus, for example, the AP sway synergy module is activated in proportion to ankle rotational input, while the vertical suspensory synergy module is activated in proportion to knee rotational input, and inhibition of the sway module by the suspensory module is provided to prevent simultaneous activation of both synergies. Such a system provides a reasonable account of the automatic first trial postural responses described above. Additionally, supraspinal processes are assumed to modulate the input-output relationships of the peripheral synergy modules in order to maintain postural stability using posturally relevant knowledge of results (e.g., sensory conflict between somatosensory and vestibular sources of information concerning the body's orientation relative to the support base and the line of gravity). Such supraspinally controlled modulation effects are presumed to occur relatively slowly, and are posited to underly the multtrial postural adaptation phenomena described above.

Task dynamics offers an attractive alternative to this hierarchical modular synergy approach. In the modular approach, synergies are canonically represented as stored output patterns, and are triggered by corresponding dis-
tinctive features of somatosensory inputs. When the problem of postural control is formulated in task dynamic terms, however, synergies need not be canonically represented anywhere; rather, synergistic patterns of muscle activity may be viewed as emergent properties of the task dynamically organized postural system. In this latter view, one may define a postural task space (see Figure 15A) in the following way, using postural control only in the sagittal plane for purposes of illustrative simplicity. This task space is modeled as a two-dimensional point attractor for which: a) the terminal device is the body's center of mass and is represented as a point mass with mass \( m_T \) equal to total body mass (note that, unlike earlier examples, this terminal device cannot in general be associated with a particular point on the linkage); b) axis \( t_2 \) is defined parallel to the line of gravity and axis \( t_1 \) is defined normal to \( t_2 \); the \( t_1 t_2 \) origin is defined by the target location of the mass center, which coincides with the mass center's initial location (assuming a corresponding posturally stable initial body configuration). The task space equations of motions are:

\[
\begin{align*}
&m_T \dddot{t}_1 + b_T \ddot{t}_1 + k_T t_1 = 0 & (26a) \\
&m_T \dddot{t}_2 + b_T \ddot{t}_2 + k_T t_2 = 0, \text{ where} & (26b)
\end{align*}
\]

the damping and stiffness parameters define point attractor topologies along each task axis. Gravity does not appear explicitly in (26b) since \( t_2 \) denotes displacement from the statically stable vertical position of the task mass in the gravitational field. In other words, \( t_2 = t_2^* - \left( \frac{m_g}{k_T} \right) \), where \( t_2^* \) corresponds to the statically stable "vertical" position of the task mass in the absence of gravity, and \( g \) denotes the acceleration due to gravity. In matrix form these equations become:

\[
\begin{align*}
&M_T \ddot{t}_1 + B_T \dot{t}_1 + K_T t_1 = 0 & (27)
\end{align*}
\]

The pattern of task spatial dynamic parameters in (27) may be transformed into body spatial form with reference to a coordinate system whose origin coincides with the center of the support base. The spatial relationships between task space and body space are illustrated in Figure 15B in which: a) the \( x_1 \) axis is defined along the anteroposterior line between the rear and front edges (denoted by open squares) of the support base, which is defined by the contact areas between the feet and ground surface; b) the \( x_2 \) axis is defined normal to \( x_1 \) at the midpoint of the support base; c) the relative orientation between task and body space is defined by the angle \( \Theta \); and d) the location of the task space origin in body space coordinates is defined by \( x_0 \). It should be noted that both \( \Theta \) and \( x_0 \) are defined by the current postural configuration, which is assumed to be statically stable, i.e., the projection of the initial location of the center of mass (task space origin) along the line of gravity will fall within the boundaries of the support base. In this regard, the task dynamic approach to vertical posture control is similar to the model proposed by Litvintsev (1972), who stated that it is likely that "the essential role in equilibrium maintenance is played by a mechanism which organizes muscular control at the various joints by parameters characterizing the general body position...(p. 590)," and that "the magnitude and the rate of deviation of the weight center projection on the support plane are input parameters for this mechanism (p. 598)." Finally, it should be noted that: a) in most daily activities we stand on horizontal surfaces and \( \Theta \) thereby usually assumes a value of zero (Figure 15C); and b) the body spatial
Figure 15. Postural maintenance task: A. Task space; B. Body space. Open boxes represent front and back edges of support base (feet). Orientation angle, $\Theta$, between task and body space is nonzero; C. Body space. $\Theta$ is zero, representing parallel orientation of $t_1$ and $x_1$; D. Postural effector system.
equations of motion derived from (27) have the same form as equation (6) in our earlier discrete reaching example.

The body spatial pattern of dynamic parameters may be transformed into an equivalent task network expression based on the joint variables of a (simplified) four-segment (foot, shank, thigh, torso), three-joint (ankle, knee, hip) effector system (Figure 15D). This task network equation has the same general form as the discrete reaching equation (8), except that the postural task network involves three joints (not two joints), and two spatial variables, defining thereby a redundant task-articulator situation (see footnote 7) and hence requiring the use of the Jacobian pseudoinverse, $J^+$, or weighted pseudoinverse, $J^\#$.

In the task dynamic framework, it is evident that consistent synergistic patterns of postural responses will occur in response to given types of destabilizing inputs. If a task network is established according to an accurate evaluation of the spatial relationships between task and body space, these postural responses will be stabilizing and compensatory. Further, they will be "immediately" accurate since they depend only upon the current limb state and the (accurately tuned) task network. In other words, synergistic responses emerge from the (tuned) postural system's underlying task dynamic organization; there is no need to invoke the notion of access to and triggering of stored canonical synergy output programs. However, the postural system can be fooled into establishing an improperly tuned task network based either on an inappropriate evaluation of the task-body space geometric relationship, or on the use of an inappropriate weighting strategy for the joints in the (redundant) postural effector system. In the former case, for example, a series of trials involving AP translation perturbations requires tuning $\bar{\omega} = 0$, since the support base is horizontal throughout the trials. If a direct rotation perturbation is unexpectedly introduced, this setting is no longer valid and the task network will shape postural responses that are inappropriate and destabilizing for the new task-body space geometry. Adaptive responses to direct rotation perturbations require setting $\bar{\omega} = \phi_1$ (where $\phi_1$ = ankle angle) in order to tune the task network appropriately. Apparently, this sort of retuning process does not occur instantaneously, but requires 3-5 trials as discussed earlier.

In the case of tunings related to effector system weighting strategies, it appears that an efficient strategy for dealing with AP translation perturbations of the foot plates is an ankle-predominant one when the feet rest directly on the plates, but a hip-predominant one when the feet rest on narrow beams. These strategies would serve to tune differentially the weighting matrices for the task network (via the weighted pseudoinverse, $J^\#$) according to the current support surface configuration. If the support surface context is changed, say, from plate to beam support, then the ankle-weighted $J^\#$ used for the plate context will be inappropriate for (or less efficient than) the new beam context. Apparently, adaptively retuning $J^\#$ to reflect a hip predominant strategy (and vice versa for hip to ankle strategy retuning) requires approximately 5-20 trials as discussed earlier.

B. Rest angle trajectories: Network coupling

1. Rest angles: final position control, trajectory formation. It was noted above (in the Topology and Dynamics section), that discrete target acquisition tasks in one degree of freedom systems (e.g., at the elbow joint)
were observed to display properties homologous to damped mass-spring systems by several investigators (e.g., Cooke, 1980; Feldman, 1966; Kelso, 1977; Polit & Bizzi, 1978; Schmidt & McGown, 1980) and had been modeled, essentially, as point attractors in an articulator dynamic sense, requiring only the setting of the final or target rest angle parameter (but see footnote 4). According to the so-called "final position control" hypothesis (e.g., Bizzi, Accornaro, Chapple, & Hogan, 1981; Kelso & Holt, 1980; Sakitt, 1980), the relative levels of neural activation of the spring-like agonist and antagonist muscle groups at a joint define an equilibrium point between two opposing length-tension curves and consequently a joint angle. It has been suggested that the transition from a given position to another may occur whenever the CNS (central nervous system) generates a signal shifting the equilibrium point between the two muscles by selecting a new pair of length-tension curves (Bizzi et al., 1981).

According to this schema, movements are, at the simplest level, transitions in posture. This simple idea is attractive because the details of the movement trajectory will be determined by the inertial and visco-elastic properties of muscles and ligaments around the joint (ibid).

However, as we discussed above (Section III), such an articulator-dynamic control scheme breaks down when more complex multijoint tasks are considered (see also footnote 4). Further, even for single degree of freedom positioning tasks, the final position control hypothesis may be incomplete. Bizzi and his colleagues (Bizzi & Abend, 1982; Bizzi et al., 1981; Bizzi, Accornaro, Chapple, & Hogan, 1982; Bizzi, Chapple, & Hogan, 1982), for example, have suggested that the rest angle trajectory is controlled in addition to final position. Thus, the final position control hypothesis predicts that elbow movements result from rapid shifts to target equilibrium points and that, consequently, steady state equilibrium positions would be achieved after a delay from muscle activity onset due solely to the dynamics of muscle activation. Bizzi, Chapple, and Hogan (1982) offer a "slowest case" approximation of 150 ms for the time taken by the net muscle force to rise within a few percent of its final value. In fact, however, these investigators showed that for movements at least 600 ms in duration, the mechanical expression of alpha motoneuronal activity reached steady state only after at least 400 ms had passed following the onset of muscle activity. Consequently, it appears that the centrally generated rest angle signal gradually changes during the movement, even in deafferented monkeys, such that the alpha motoneuronal activity defines "a series of equilibrium positions, which constitute a trajectory whose end point is the desired final position" (ibid). Finally, it should be noted that Bizzi et al. (1981) interpret their observations as implying the existence of trajectory plans or programs to account for the observed time courses of rest angle movement as well as the final rest angle position.

The control law version of task dynamics is unable to account for these data for two reasons. First, there is no parameter corresponding to rest angle in the single degree of freedom case or rest configuration in the multi-degree of freedom case. Second, the control law version assumes that \( \dot{q} \) and \( \ddot{q} \) (real arm state) are perceived proprioceptively, that \( \dot{g} \) and \( \ddot{g} \) (model arm state) equal the real arm's state, and that control laws are specified according to the currently perceived real arm's state. In the "deafferented" case,
in which the current $\mathcal{Q}$ and $\dot{\mathcal{Q}}$ are unavailable, the control laws are undefined and (coordinated) motion is not possible. Given the above "trajectory formation" data of Bizzi and colleagues, if task dynamics is to be applied in these situations, the control law version must be amended to generate coordinated movements in deafferented preparations and to include a rest configuration parameter (which, of course, must evolve autonomously during the movement according to task–dynamic constraints). Although in preliminary form, we believe a network coupling version of task dynamics satisfies these requirements and provides a more biologically plausible task dynamic account of skilled movements.

2. Network Coupling. The network coupling method (outlined in Figure 16) involves shaping articulator dynamics according to task-specific dynamical constraints and may closely approximate a biological style of coordination and regulation. Briefly, the network coupling method involves interpreting the observed skilled motion of an effector system to be the observable "output" of an articulator network that comprises, however, only one half of a task specific action system. The complete action system consists of the mutually or bidirectionally coupled task (output variables: $\mathcal{Q}$, $\dot{\mathcal{Q}}$, $\ddot{\mathcal{Q}}$, etc.) and articulator (output variables: $\mathcal{Q}$, $\dot{\mathcal{Q}}$, $\ddot{\mathcal{Q}}$, etc.) networks. Thus, for the multidegree of freedom discrete reaching task described earlier, this method involves: a) treating the task network defined in equation 8 as a system for intrinsic pattern generation that is specified for a given task and actor–environment context, and that does not require peripheral input for its operation; b) defining the articulator network corresponding to an actual arm by the following version of equation (11):

$$\ddot{\mathcal{Q}} + M_A^{\ddot{\mathcal{Q}}} \dot{\mathcal{Q}}_p + M_A^{\Gamma} \dot{\mathcal{Q}} + M_A^T A \mathcal{A} A \dot{\mathcal{Q}} + M_A^\Gamma A A_a = \mathcal{Q}$$  \hspace{0.5cm} (28)

![Diagram](image-url)  

Figure 16. Overview of information flow in network coupling version of task dynamics.
and c) using the task network to both actuate and modulate the articulator network, while using the articulator network to modulate the task network.

More specifically, the network coupling method begins by using the \( \mathcal{Q} \) output of the task network as the \( \mathcal{Q}_o \) input ("rest configuration," i.e., \( \mathcal{Q}_o = \mathcal{Q} \)) for the articulator network. However, since the task and articulator networks are potentially independent, one cannot simply assume identical task and arm network states (as in the control law approach). Rather, we make the less stringent assumption that real arm and model arm states are "close," i.e., that \( \mathcal{Q} - \mathcal{Q}_o = \Delta \mathcal{Q} \) and \( \mathcal{Q} - \mathcal{Q}_o = \Delta \mathcal{Q}_o \) are "small." Therefore, the constraint relationships for \( B_A, K_A \), and \( \mathcal{J}_{A_B} \) are defined in a more approximate sense than those in equation (13) (see Appendix C for details):

\[
\begin{align*}
B_A &= [M_A J^{-1} B_B J] \mathcal{Q}_o = \mathcal{Q} \\
K_A &= [M_A J^{-1} B_B J] \mathcal{Q}_o = \mathcal{Q} \\
\mathcal{J}_{A_B} &= [M_A J^{-1} B_B J] \mathcal{Q}_o = \mathcal{Q} - \mathcal{Q}_o 
\end{align*}
\tag{29a}
\tag{29b}
\tag{29c}
\]

These sets of "driving" constraints (\( \mathcal{Q}_o = \mathcal{Q} \)) and "modulating" constraints (equations 29) comprise the "efferent" aspect of our coupled-network action system. With these constraints, the articulator network becomes (statically) stable about the current rest configuration, with stiffness and damping properties defined relative to task space axis directions.

However, a coupled-network action system involves bi-directional coupling and hence an "afferent" aspect as well. This pattern of afferentation serves to modulate the activity of the task network on the basis of both relative angular displacement (\( \Delta \mathcal{Q}_o = \mathcal{Q}_o - \mathcal{Q}_o \)) and relative angular velocity (\( \Delta \dot{\mathcal{Q}}_o = \dot{\mathcal{Q}} - \dot{\mathcal{Q}} \)) coupling terms defined by \( \alpha \Delta \mathcal{Q}_o \) and \( \beta \Delta \dot{\mathcal{Q}}_o \), respectively, where \( \alpha \) and \( \beta \) are constant scalar coupling coefficients. This type of coupling, which is proportional to differences between corresponding sets of state variables, is called diffusive coupling (e.g., Rand & Holmes, 1980). The modulated task network is then described by the following amended version of equation (8):

\[
\ddot{\mathcal{Q}} + J^{-1} M_B B_B J \ddot{\mathcal{Q}} + J^{-1} M_B B_B A_B(\mathcal{Q}) + J^{-1} \mathcal{J}_{A_B} + \alpha \Delta \mathcal{Q}_o + \beta \Delta \dot{\mathcal{Q}}_o = \mathcal{Q},
\tag{30}
\]

where by assumption \( \Delta \mathcal{Q}_o \) and \( \Delta \dot{\mathcal{Q}}_o \) are assumed "small." The effects of these coupling terms on system behavior are to reduce the size of \( \Delta \mathcal{Q}(= -\Delta \mathcal{Q}_o) \) via \( \alpha \Delta \mathcal{Q}_o \) coupling and to reduce the size of \( \Delta \dot{\mathcal{Q}} \) via \( \beta \Delta \dot{\mathcal{Q}}_o \) coupling, thereby promoting an in-phase (vs. anti-phase) one-to-one relationship between real and model arm motions. It should be noted that equation (30) reverts to equation (8) when \( \Delta \mathcal{Q}_o \) and \( \Delta \dot{\mathcal{Q}}_o \) equal zero (i.e., there is perfect mutual tracking of the real and model arms) or when the afferent coupling is disengaged (i.e., peripheral feedback is eliminated and the system is "deafferented") by setting \( \alpha \) and \( \beta \) to zero. Further, one should note that, even when deafferented, the model arm is governed by the task network equation (30) and hence \( \mathcal{Q} \) is capable of coordinated (although probably degraded) motion due to "internal feedback" of the model arm's current state within the task network. Here, internal feedback is used in the sense of Evarts (1971) to indicate information "arising from structures within the nervous system" as opposed to peripheral information from proprioceptive sources in the (real) limbs. Finally, although the operation of the coupled action system involves regulating \( \Delta \mathcal{Q}(= -\Delta \mathcal{Q}_o) \) and
\( \Delta \theta (\approx \hat{\theta}) \) to be "small," this is not solely a function of \( \hat{\theta} \) position control requirements per se but also serves to validate the "small" relative displacement and velocity assumptions used for the real arm control matrices specified in relationship (29).

In summary, the network coupling version of task dynamics may provide a more biologically relevant sensorimotor control scheme than does the control law version. For single degree of freedom positioning tasks, it provides a rational account of the centrally specified rest angle's trajectory for these tasks without needing to invoke an explicitly preplanned representation of that trajectory. Rather, the rest angle trajectory evolves, even in the deafferented case, as an ongoing function of the underlying task dynamics. Similarly, when applied to discrete planar reaching tasks of 2-joint arms, the rest configuration trajectory will evolve so that the hand should move in a quasi-straight-line from initial to final position. Finally, when applied to cyclic spatial movements of a multijoint arm, the network coupling approach shares certain features with recent work on locomotion (cf. Grillner, 1981, for review). Investigators in this field assume the existence of innate, endogenous, cellular networks that are: a) capable of driving the limbs according to the locomotor task without requiring peripheral information; yet b) can be modulated—in phase dependent ways—by this same peripheral input (e.g., Forssberg, Grillner, & Rossignol, 1975). Task networks may be interpreted as the abstract, learned analogs of such concretely defined, innate networks. Thus, from a task dynamic perspective, the origins of task networks lie in the active discovery and specification processes that occur during skill learning. Once acquired, their operation is tailored to (tuned by) currently perceived task demands and the actor–environment spatial context.

References


Appendix A (Equation 7)

The body spatial variables \((\tilde{x}, \tilde{y}, \tilde{z})\) of equation (6) are transformed into the joint variables \((\tilde{q}, \tilde{\dot{q}}, \tilde{\ddot{q}})\) or a massless arm model using the following kinematic relationships:

\[
\begin{align*}
\tilde{x} &= \tilde{\tilde{x}(\tilde{q})} \\
\tilde{\dot{x}} &= J(\tilde{q})\tilde{\dot{q}} \\
\tilde{\ddot{x}} &= J(\tilde{q})\tilde{\ddot{q}} + (dJ(\tilde{q})/dt)\tilde{\dot{q}} \\
&= J(\tilde{q})\tilde{\ddot{q}} + V(\tilde{q})\tilde{\dot{q}}_p, \text{ where}
\end{align*}
\]

\(\tilde{x}(\tilde{q})\) = the current body spatial position vector of the terminal device expressed as a function of the current model arm configuration;

\[
\tilde{\ddot{q}} = \begin{bmatrix} l_1 \sin \phi_1 + l_2 \sin(\phi_1 + \phi_2) \\
-l_1 \cos \phi_1 - l_2 \cos(\phi_1 + \phi_2) \end{bmatrix}^T;
\]

\(\tilde{\dot{q}}_p = [\tilde{\dot{q}}_1^T, \tilde{\dot{q}}_2^T]^T\), the current joint velocity product vector;

\(J(\tilde{q})\) = the Jacobian transform matrix;

\[
J(\tilde{q}) = \begin{bmatrix}
(l_1 \cos \phi_1 + l_2 \cos(\phi_1 + \phi_2)) & l_2 \cos(\phi_1 + \phi_2) \\
(l_1 \sin \phi_1 + l_2 \sin(\phi_1 + \phi_2)) & l_2 \sin(\phi_1 + \phi_2)
\end{bmatrix};
\]

\(V(\tilde{q})\) = a matrix resulting from rearranging the terms of the expression

\(\tilde{\ddot{q}} - (dJ(\tilde{q})/dt)\tilde{\dot{q}}\) in order to segregate the joint velocity products into a single vector \(\tilde{\dot{q}}_p\):

\[
V(\tilde{q}) = \begin{bmatrix}
-l_1 \sin \phi_1 - l_2 \sin(\phi_1 + \phi_2) & -2l_2 \sin(\phi_1 + \phi_2) - l_2 \sin(\phi_1 + \phi_2) \\
l_1 \cos \phi_1 + l_2 \cos(\phi_1 + \phi_2) & 2l_2 \cos(\phi_1 + \phi_2) l_2 \cos(\phi_1 + \phi_2)
\end{bmatrix}.
\]

Making these substitutions into (6) and rearranging, we get equation (7):

\[
M_B J \tilde{\ddot{q}} + B_B J \tilde{\dot{q}} + K_B A\tilde{x}(\tilde{q}) = -M_B V\tilde{\dot{q}}_p, \quad (A4), (7)
\]

It should be noted that since \(A\tilde{x}\) in equation (6) is not assumed "small," the differential approximation \(d\tilde{x} = J(\tilde{q})d\tilde{q}\) is not justified and, therefore, equation (A1) was used instead for the kinematic displacement transformation into model arm variables.
Appendix B (Equation 9)

One may derive using a Lagrangian analysis (see, for example, Saltzman, 1979, for details) the passive mechanical equations of motion for the 2-segment arm (frictionless, no gravity) described in the text:

\[ M_A \ddot{q} + S_A \dot{q} = 0, \quad \text{where} \quad (A1), \quad (9) \]

\[ M_A = M_A(q), \text{ the } 2 \times 2 \text{ acceleration sensitivity matrix with elements } q_{ij}, \text{ where} \]

\[ q_{11} = m_2(l_1^2 + (1/3)l_2^2 + l_1l_2\cos \theta_2) + m_1(1/3)l_1^2 \]
\[ q_{12} = m_2((1/3)l_2^2 + (1/2)l_1l_2\cos \theta_2) \]
\[ q_{21} = q_{12} \]
\[ q_{22} = (1/3)m_2l_2^2; \]

\[ S_A = S_A(q), \text{ a } 2 \times 3 \text{ matrix with elements } s_{ij}, \text{ resulting from} \]

\[ \text{rearranging the terms of the coriolis and centripetal torque terms} \]

\[ \text{in order to segregate the joint velocity products into a single} \]

\[ \text{vector } \dot{\bar{q}}_p, \text{ where} \]

\[ s_{11} = 0; \quad s_{12} = -m_2l_1l_2\sin \theta_2; \quad s_{13} = (1/2)s_{12} \]
\[ s_{21} = -s_{13}; \quad s_{22} = 0; \quad s_{23} = 0 \]

Appendix C (Equation 15)

I) \( K_A \). We begin with the expression \[ M_A^{-1}K_A \dot{q} = \dot{q} \] from equation (11). Since we assume that \( \dot{q} \) is "small," we are justified in making the differential approximation:

\[ [M_A^{-1}K_A \dot{q}]_{\dot{q}} = [M_A^{-1}K_A \dot{q}]_{\dot{q}} \]

\[ \text{where} \quad (C1) \]

\[ dx = x(q) - x(q_0) \text{ denotes the differential body space displacement between} \]

\[ \text{the terminal devices of the real (articulator network) and model (task network) arms, and } \]

\[ \text{[M}_A^{-1}K_A J^{-1}]_{\dot{q}} \text{ denotes the articulator stiffness pattern governing the real arm's responses to small displacements about } x(q_0) = \dot{q}. \]

The body spatial stiffness responses of the model arm specified by task dynamics for (possibly) large scale displacements \( A\bar{x}(\dot{q}) = x(q) - x(q_0) \) from the reaching target \( x \) are governed by the spatial restoring force term \[ [J^{-1}M_B^{-1}K_B \bar{x}(q)]_{\dot{q}} \text{ in equation (8). Assuming that the model } (\dot{q} = \dot{q}_0) \]

\[ \text{and real } (\dot{q}) \text{ arm configurations are "close," we compare the stiffness expressions and define the following constraint relationship:} \]

\[ K_A = [M_A^{-1}K_A J^{-1}]_{\dot{q}_0} = \dot{q} \quad (C2) \]
This relationship specifies that stiffness responses of the real arm to small $\Delta \theta$ perturbations will be defined according to task space axis directions and task space stiffness weightings.

II) $B_A$, $J_A$. Assuming that both $\Delta \theta$ and $\dot{\Delta} \theta$ are "small," one may use equations (8) and (14b) to define the following constraint relationships:

\[ B_A = [M_A J^{-1} M_B B J] | \dot{\theta} = \dot{\phi} \] \hspace{1cm} (C3)

\[ J_A = [M_A J^{-1} V_S A] | \eta = \dot{\eta} \] \hspace{1cm} (C4)
Footnotes

1 An effector system is the set of limb segments or speech organs used in a given action; a terminal device or end effector is the part of a controlled effector system that is directly related to the goal of a performed action. Thus, in a reaching task, the hand is the terminal device and the arm is the effector system; in a cup-to-mouth task, the grasped cup is the terminal device and the hand-arm system is the effector system; in a steady state vowel production task, the tongue body surface is the terminal device and the jaw-tongue system is the effector system.

2 Different systems may have different types of escapements. For example, van der Pol and Rayleigh oscillators have related escapement terms that are continuous functions of the systems' states; the pendulum clock's escapement term is a discontinuous function of the system's state, injecting a pulse of energy at one or two discrete points in the cycle.

3 For a task in which an arm is nonredundant, the number of controlled spatial variables for the terminal device is equal to the number of controlled joint angular variables for the arm. Hence the inverse kinematic transformation from spatial motions of the terminal device to corresponding arm joint angular motions is determinate. For a task in which the number of joint variables exceeds the number of spatial variables, this transformation is indeterminate and the arm is redundant. For redundant arms, one may specify the inverse kinematic transformation by: a) "freezing" the extra joints in the arm; b) adding extra controlled spatial variables to the task description; or c) specifying optimality criteria to be satisfied for the joint variables during the movement.

4 Indeed, herein lies an important difference between the various versions of the mass-spring model (or equilibrium point hypothesis for discrete targeting behavior). In one widespread view that is restricted to single degree of freedom motions, muscles are represented by a pair of springs acting across a hinge in the agonist-antagonist configuration. The final equilibrium point is established by selecting a set of length-tension properties in opposing muscles (e.g., Bizzi, 1980; Cooke, 1980; Kelso, 1977). This view, at best, may work for deafferented muscle, but, as pointed out by Fel'dman and Latash (1982, p. 178) it is inadequate for muscles in natural conditions. Moreover, as we have taken pains to point out, it does not work for complex, multivariable tasks. An alternative view, which we elaborate upon here, is that the parallel between a single muscle and a spring is not a literal one. Instead, the mass-spring model is better viewed as a model of equifinality or motor equivalence: it is this abstract functional property that particular behaviors share with a mass-spring system (Kelso, Holt, Kugler, & Turvey, 1980; Kelso & Saltzman, 1982). In short, the former, articulator dynamic version is a hypothesis about a physiological mechanism whose shortcomings have been noted (Bizzi, Accornero, Chapple, & Hogan, 1982; Fel'dman & Latash, 1982). The latter, abstract dynamic version refers to a complex system, and is a hypothesis about behavioral function.

5 Currently, our task-dynamic formulation does not include precision force control tasks. It can be easily adapted for tasks that demand particular motion patterns along a surface and only approximate control of the force exerted by the terminal device normal to the surface (e.g., polishing a car, erasing a blackboard). The approach can also be adapted for precision force control tasks, however, as demonstrated by Hogan and Cotter (1982).
Mason (1981; see also Raibert & Craig, 1981) has formalized a related geometrical description for manipulator contact tasks in which different tasks are characterized by distinct generalized surfaces in a constraint space. In this task-specific constraint space, the task degrees of freedom are partitioned into those associated with either position or contact force control, respectively, during performances of the associated task. Such an approach requires, however, explicit task- and context-specific position and force trajectory plans for the task's terminal device. In contrast, the task dynamic approach requires no such explicit trajectory plans, due to the task-specific dynamical topologies defined for the task-space degrees of freedom. In our formulation, then, task-appropriate terminal device trajectories are emergent properties implicit in the corresponding underlying task-dynamic organizations.

In redundant task-articulator situations (see footnote 3), $J^{-1}$ is not defined and the Jacobian pseudoinverse ($J^+$) or weighted Jacobian pseudoinverse ($J^*$) may be used (Benati et al., 1980; Klein & Huang, 1983; Whitney, 1972). Using $J^*$ provides an optimal weighted least squares solution for the differential transformation from spatial to joint motion variables. If this weighting is task dependent, then $J^*$ would be both task- and configuration-dependent. For example, if a three-joint arm is used to position the fingertip in a spatially planar reaching task, different weightings would correspond to different arm joint motion strategies. One weighting might correspond to a predominantly shoulder motion strategy, while a second weighting might specify a predominantly elbow motion strategy, etc. In such cases, elements of the weighting matrices used for the corresponding weighted Jacobian pseudoinverses define a further set of tuning parameters for the task network.

As we demonstrate via simulation (in the Trajectory Shaping section) in the case of our task space point attractor reaching example, it may be possible to ignore the velocity product torque terms, and therefore omit $\tau_A$ from equation (12), yet still arrive at the desired target via quasi-straight line hand trajectories. In fact, reach trajectories generated without such correction appear more similar to experimentally observed trajectories than ones generated with "perfect" velocity product torque correction.

The desirability of using such scaling coefficients was pointed out by Mason (1981). In addition to using them to ensure dimensional homogeneity, Mason showed that different values could be used to provide correspondingly different weightings of rotational vs. linear aspects of task performances. However, since the task dynamic approach uses relative task axis stiffness weightings for this purpose, the value of $\rho$ was simply set to 1.0 in our treatments.

For task spaces not defined by point attractors along each task axis, however, equation 29a will no longer hold. For example, if a given task has a limit cycle organization for one task axis, and therefore a nonlinear damping term, the $B_T$ and hence $B_A$ matrices will reflect only the linear negative part of this damping. If $B_A$ were used in equation 29a, the articulator network should be highly unstable. In such cases, however, one might simply choose $B_A$ to make the articulator network stable about $\theta_0$, given the $K_A$ specified in equation 29b.